The Role of Uncertainty in Jobless Recoveries

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Abstract

The three most recent downturns, in contrast with other post-war recessions, are characterized by slow recoveries in employment despite positive economic growth. I find that recent recoveries coincide with high uncertainty about economy-wide corporate profits at a time when output begins to rebound, a pattern that was not observed in the earlier recessions. To examine the role of uncertainty in jobless recoveries, I develop a dynamic stochastic general equilibrium model with search and matching frictions in the labor market and an intensive labor margin. The model is driven by productivity and time-varying volatility shocks. The uncertainty agents face is captured by time-varying volatility. Labor market search frictions generate costly labor adjustment. When an uncertainty shock hits the economy, firms reduce the number of vacancies posted because they are reluctant to make costly adjustments along the extensive margin. Instead, firms require more effort from their employees. This, along with positive productivity shocks, can result in jobless recoveries. I calibrate the model and show that, with the addition of uncertainty shocks, this model can replicate recent episodes of jobless recoveries.

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1 Introduction

One of the most stable macroeconomic relationships in the post-war era has been the co-movement of output and employment. Indeed, the inverse relationship between output and unemployment was first quantified in Okun (1962), and this robust empirical regularity has come to be known as Okun’s law. Moreover, conventional macroeconomic models, including search-and-matching models, predict strong co-movement between output and employment. All of these make the anemic job growth following the last three recessions particularly puzzling.

In this paper, I explore the role of uncertainty in jobless recoveries. I observe that recent downturns are characterized by high levels of uncertainty about economy-wide corporate profits at a time when output began its recovery; this is in contrast with earlier recessions where uncertainty fell after the official end of the recession. This suggests that high uncertainty could hold back employment recovery despite positive output growth. Furthermore, I show that the peaks of uncertainty coincide with troughs of employment following recent recessions—which suggests that when uncertainty about the state of the economy begins to fall following a recession, employment begins to climb. These findings relate our intuition that managers act more cautiously when they are unsure of their future prospects to the anemic employment growth in recent recoveries.

To account for these empirical findings, I develop a dynamic stochastic general equilibrium model that features search-and-matching frictions in the labor market and an intensive labor margin. The main driving force of the model is a productivity process. I capture the notion of uncertainty that agents face with time-varying volatility to the productivity shocks. When an uncertainty shock arrives, firms become more unsure of their future prospects. Labor market search friction introduces costly labor adjustment; given that firms want to avoid costly mistakes, they turn to the intensive margin by requiring more efforts from their employees and reduce the number of vacancies posted, resulting in jobless recoveries.

To evaluate the quantitative properties of the model, I first solve it using third-order perturbation methods; third-order solutions allow me to examine the dynamics of the model perturbed by volatility shocks. I then solve for two stochastic processes—productivity and its volatility—that allow me to match the U.S. employment and output. I ask two main questions: (1) Does the model show a lack of jobless recoveries once I counterfactually remove the uncertainty process? and (2) Does the implied volatility process behave as a reasonable proxy of uncertainty? The answer to both questions is yes. Moreover, the model-implied labor input (combining both the extensive and the intensive margins) mirrors U.S. aggregate hours, suggesting that the model does not rely on

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1In the context of business cycle literature, the main driving force of the model is typically interpreted as technology or productivity, and I will refer to it as such for the remainder of this paper. It should be noted, however, in a search-and-matching framework, firms’ employment decision might be driven by more than productivity alone. In fact, the driving force in Mortensen and Pissarides (1994) is the marginal revenue product of labor which might better represent the value of a worker to the firms.

2See Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez, and Uribe (2011) for a discussion on why a higher-order solution method is required when one wishes to examine the impact of second-order perturbations.

3This exercise implicitly and naively assumes that productivity and its volatility are the only determinants of output and employment, which is most certainly not the case in the real world. This exercise is nonetheless informative because while it may yield counterfactually large uncertainty shocks in order for the model to match the data, it allows us to focus on the role of uncertainty plays without being distracted by other features.
unrealistic intensive margin adjustment to match the data, which further strengthens the results of this model. Thus, the conclusion we can draw from the exercise is that uncertainty does indeed play a key role in recent episodes of jobless recoveries; without it, the model fails to replicate this feature of the data.

This paper is related to many strands of literature. Beginning with pioneering work by Merz (1995) and Andolfatto (1996) in embedding labor search friction in business cycle models, papers such as Cooley and Quadrini (1999), den Haan, Ramey, and Watson (2000), Krause and Lubik (2007), and Gertler, Sala, and Trigari (2008), among others, have built upon that foundation and incorporated search-and-matching in many different, and ever more complex, macroeconomic models in efforts to draw upon the rich insights search-and-matching models offer.

This paper is also related to the strand of literature that examines the role of uncertainty as a driving force of economic activities. For example, Bloom (2009), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012), and Bachmann and Bayer (2011) study the extent to which aggregate and idiosyncratic uncertainty shocks affect the dynamics of the economy through a wait-and-see mechanism. Fernández-Villaverde, Guerrón-Quintana, Keuster, and Rubio-Ramírez (2011) and Baker, Bloom, and Davis (2013) study the effects of policy uncertainty. While the former estimates fiscal policy uncertainty and the latter constructs an index which includes components such as forecast disagreements, news media references, and the number of expiring tax code provisions, both studies find policy uncertainty shocks depress economic activities. In addition, Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez, and Uribe (2011) find that volatility shocks to the interest rate at which small open economies borrow have a significant impact on the business cycle outcomes of these economies.

This paper combines the approaches of these two strands of literature and focuses its attention on the interaction of uncertainty and the employment dynamics in a frictional labor market in order to shed light on jobless recoveries.

Lastly, this paper is related to other works on jobless recoveries. Among others, Andolfatto and MacDonald (2004) look at how technological diffusion can contribute to slow job growth in a recovery; Shimer (2010) argues the transitional dynamics of an economy that features rigid wages and has lost some of its capital stock resembles that of a jobless recovery; van Rens (2004) shows that by allocating more workers from organizational to productive tasks, firms can increase output for a time before the need to hire new workers arises. Without ruling out the conclusions of these studies, this paper takes a different approach by incorporating time-varying volatility to the productivity process; it seeks to understand the relationship between uncertainty and the macroeconomic labor market.

In terms of its approach to explain jobless recoveries, this paper most closely resemble works by Bachmann (2011) and Schaal (2011). The former features an intensive labor margin with non-convex adjustment costs to the extensive margin; the latter incorporates a permanent uncertainty shock into a search-and-matching model with heterogeneous firms. Bachmann (2011) argues that, after a mild recession where a firm would have retained most of its workers, the need for new

\[\frac{\text{It should be noted that the goal of this paper is not to address whether uncertainty shocks drive business cycles, but to examine their role in recent episodes of jobless recoveries.}}{4}\]
workers is low in an environment where firms have access to an intensive labor margin as an input of production. This would explain the sluggish employment response. This argument might be unsatisfactory, however, in explaining the most recent recession, which ranks as one of the longest and the most severe recessions in the post-war era. This paper improves upon that dimension by incorporating the notion of uncertainty. Schaal (2011), contemporaneously developed with this paper, shows that a one-time increase to the standard deviation of the innovations to the idiosyncratic productivity in conjunction with a productivity shock can produce the joint dynamics of high unemploy-ment and positive output growth following the Great Recession. By focusing on aggregate uncertainty and utilizing the search model to generate costly labor adjustment, this paper complements Schaal (2011) by showing that, with dynamic uncertainty and an intensive margin, it can successfully replicate all three episodes of jobless recoveries. The parsimonious nature of this model allows future researchers to easily adopt the framework in a wide class of macroeconomic applications.

The rest of the paper is organized as follows. Section 2 presents the relevant empirical facts, including the relationship between uncertainty and employment. Section 3 lays out the model. Section 4 discusses the calibration strategy and details the computational strategy. Section 5 presents the results. Section 6 concludes.

2 Empirical Facts

2.1 Employment and Output

According to the establishment-level survey data from the Bureau of Labor Statistics, in the 4 NBER recoveries between 1969 and 1990, total non-farm payroll expanded above its level at the output trough 1.29 quarters from the end of the recession. That relationship disappeared, however, in the three most recent recessions. It took 6, 10, and 8 quarters before employment recovered during the 1991, 2001, and 2009 recoveries respectively.

This differential behavior of employment prior to and post-1991 is clearly seen in Figure 1. The top panel of Figure 1 plots real GDP’s recovery path while the bottom panel plots total non-farm payroll. Log deviation from either GDP’s or employment’s level at output trough is on the y-axis; and quarters after the trough is on the x-axis. Employment climbed immediately in the pre-1991 recoveries. However, in 1991 employment trough lagged output trough by 2 quarters before it finally recovered by the sixth quarter. In 2001, employment bottomed out 6 quarters after output, and it was not until the tenth quarter of the recovery it finally reached its level at the beginning of the recovery. In the most recent recession, employment dropped by a staggering 1.3% in the first three quarters of the recovery; it was not until the second quarter of this year employment caught

5Troughs refer to NBER’s business cycle troughs. This is based on quarterly average of monthly figures. Note that the start date is chosen to match the Survey of Professional Forecasters which I will describe in detail later. If I include the recessions ending in 1954, 1958, and 1961, the average time of employment recovery is 1.29 quarters.
6Figure 1 replicates Figure 1 in Bachmann (2011) with the addition of the 2009 recovery.
7The jobless recovery pattern persists if one plots other broad measures of employment such as total non-farm business employment or employment from the household surveys.
8The behaviors of individual recessions are similar to the average shown here.
Figure 1: Real GDP and total non-farm payroll. Log-deviation from output and employment levels at NEBR trough is plotted on the y-axis; quarters from trough is on the x-axis.

up to its 2009 Q2 figure, a full 8-quarter lag.

This pattern of jobless recovery remains when I examine the cyclical components of output and employment. Figure 2 shows the log deviations of real GDP and total non-farm payroll from its Hodrick-Prescott trend.\(^9\) While the relatively weak GDP growth after the 1991 and 2001 recessions might have contributed to the anemic employment growth, the cyclical component of employment did not recover until the 14th and 15th quarter for the 1991 and 2001 recessions, well after the cyclical trough of GDP. As for the 2009 recession, the drop in cyclical employment is much larger than what was typical in pre-1991 recoveries before it recovered 4 quarters into the recovery.

2.2 Uncertainty

Why should we care about uncertainty? Vistage International, an organization that trains business managers, produces Vistage CEO Confidence Index from a quarterly survey of over 1,600 CEOs of small- to medium-sized companies. Based on the seven publicly available surveys on its website, “economic uncertainty (concern for local and national economy)” consistently gathers over 30% of the responses as “the most significant business issue [the CEOs] are facing today.”\(^{10}\) Moreover,

\(^9\)I choose a smoothing parameter of 1,600 as it is typical for quarterly data. The pattern is robust to different smoothing parameters, such as 10\(^5\) used in Shimer (2005), as well as using a band-pass filter (6,32).

\(^{10}\)The seven available surveys are from 2010 Q1 to 2011 Q3. The URL is http://www.vistage.com/media-center/confidence-index-archive.aspx. The value for economic uncertainty ranges from a low of 29% in 2011 Q1 to a
Figure 2: Real GDP and total non-farm payroll, HP filtered. The $y$-axis plots the deviation of the cyclical components of log output and log employment from their levels at the NBER troughs; quarters from trough is on the $x$-axis.
in the 2010 Q3 survey, “Among the uncertainties in today’s marketplace, which concerns you the most?” “Economic outlook” was the number one concern at 59% among the 1,845 respondents, more than 30% higher than the second highest concern, “taxes,” at 26%. Lastly, in the 2011 Q2 survey, “Future demand” and “worries about the U.S. economic outlook” are cited among the top factors (28% and 26% respectively) “[that are] impacting [the CEOs’] hiring strategy the most.”

It is clear that these surveys highlight the importance of uncertainty in the minds of business managers and suggest that, to better understand the labor market, it is essential to incorporate time-varying volatility into the model.

To delve deeper into uncertainty and to see how recent recoveries are different from the earlier recoveries, I turn to the Survey of Professional Forecasters (SPF) conducted by the Federal Reserve Bank of Philadelphia. Every quarter the SPF gives a panel of forecasters a list of economic variables such as GDP, employment, and inflation rate, and ask the panelist to forecast the variables’ values for different time horizons up to six quarters into the future. I focus on the forecasts of economy-wide corporate profits. Corporate profits is a logical starting point given that I wish to study firms’ employment decisions; after all, if a manager does not expect her firm to earn a positive profit in the foreseeable future, it is likely she would not hire new workers either. Moreover, forecasts of corporate profits embody both future demand and economic outlook cited by the CEOs in the Vistage surveys as their main concerns.

Based on individual response, I take the one-quarter-ahead forecasts of corporate profits and compute the log difference of the 75th and the 25th percentile forecasts. I use this measure of disagreement as a proxy for the level of uncertainty in the economy; that is, if the 75th and the 25th percentile forecasters hold a general consensus about the future prospects of the economy, their forecasts should be similar and the disagreement would be small; on the other hand, if forecasters have vastly differing opinions, the disagreement would be large.

Figure 3 plots three series—HP de-trended output, HP de-trended employment, and uncertainty. The first thing to notice in Figure 3 is how, in the four recoveries prior to 1991, the high of 40% in 2011 Q3. The other options are: rising energy cost; growth (growing too quickly); rising health care costs; political uncertainty; staffing (finding, hiring, retaining, and training); growth (growing too slowly); financial issues (finance, cash flow, profitability); and other. The second most significant concern is financial issues, which hovers around 16% throughout 2010.

11The other choices are: results of the term election; and other.
12“My hiring strategy has not been impacted” also has 28%. The other two choices are “uncertainty around the country’s fiscal environment” and “uncertainty around the country’s regulatory environment” which combine for 18%.
13For example, suppose in 2010 Q2 forecaster Alice predicted the corporate profits in 2010 Q3 to be $1.1 trillion and she was the 75th percentile forecaster; forecaster Bob, who was the 25th percentile forecaster, predicted $1 trillion. The measure of disagreement for 2010 Q2 would be $log \left( \frac{1.1}{1} \right) \approx 0.095.
14This paper follows the lead of many in the literature that use forecast disagreement as a proxy for uncertainty. See Holland (1993) for an extensive list of earlier literature that uses forecast disagreement in this capacity; Fuss and Vermeulen (2008), Popescu and Smets (2010), and ? are some of the recent studies that do the same. It is true that the disagreement in point estimates such as ones given for corporate profits in the SPF does not represent forecasters’ uncertainty. However, there is evidence suggesting that this is an acceptable proxy—among others, Zarnowitz and Lambros (1987) and Bomberger (1996) find a stable and significant relationship between inflation disagreement and uncertainty; Giordani and Söderlind (2003) confirms the view that disagreement is a good proxy for uncertainty.
15To remove high frequency fluctuations, Figure 3 plots the one-quarter centered moving average of the corporate profit forecast disagreement as uncertainty. The correlation between the original forecast disagreement and the centered moving average series is 0.77. Note that moving average makes the extreme points less extreme. Given that
Figure 3: HP de-trended log GDP and employment with corporate profit forecast disagreement. The left $y$-axis is log deviation from HP trend. The right $y$-axis is the log-difference of the 75th percentile forecast and the 25th percentile forecast of the one-quarter-ahead economy-wide corporate profits. The shaded areas are NBER recessions.
cyclical troughs of employment (blue dashed line with $\times$s) follow closely the cyclical troughs of output (green line with dots); whereas post-1991, they lag behind output by several quarters—these correspond to the jobless recoveries described above.

Secondly, in the three most recent recoveries, uncertainty about economy-wide corporate profits remains high much longer into recent recoveries; this is in contrast with earlier recoveries where uncertainty falls after the official end of the recession. Just as importantly, unlike the pre-1991 recoveries where uncertainty (red solid line) is either already low or is dropping contemporaneously with the turning points of output, the turning points of output are accompanied by either a high level of uncertainty or an upward jump in uncertainty. Lastly, one can observe that the peaks of uncertainty coincide with the troughs of employment—that is to say, when uncertainty about the states of the economy begins to fall, employment begins to climb. This finding relates our intuition that managers act more cautiously when they are unsure of their future prospects to the anemic employment growth in recent recoveries.

3 Model

This section describes the model used to examine the role of uncertainty in jobless recoveries. The basic building blocks of this model are: (1) a search-and-matching labor market; (2) households; (3) firms; and (4) a government.

Households make consumption and investment decisions. I follow Merz (1995) and Andolfatto (1996) in assuming that each household consists of an infinite number of members, some of whom are employed and some are not. There is perfect risk-sharing among household members so each member consumes the same amount as the others. Moreover, while the extensive margin is supplied inelastically, I assume that employed workers incur disutility from exerting effort. Employed workers receive wage payment while unemployed workers receive unemployment insurance from the government, which levies a lump-sum tax to finance the program. In addition to labor income, households receive rental income from supplying firms with capital as well as firms’ profits in the form of dividend payments.

Firms make production decisions. The inputs are capital and labor. Firms rent capital from a competitive market taking the rental rate as given. They post vacancies to attract workers in order to expand the extensive margin of the labor input. Relationships are formed when a vacancy is matched up with a job seeker. Firms can expand their production through an intensive labor margin I call hours. The hours and wage schedule (wage depends on hours worked) are negotiated between a firm and its workers. Firms are owned by households; any profits go to the households as dividend payments.

Peaks are what interests us, my point remains with the actual, non-moving-averaged series. I have experimented with different measures of disagreement—different time horizons, expected profit growth instead of level difference, etc.—they draw the same inference I make here.

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However, it is important to keep in mind that the intensive margin represents a broader, and often unobservable, measure of effort on the part of the workers.
I now provide the details of my model, starting with the labor market.

### 3.1 Labor Market

There is a unit mass of workers in the economy. At the beginning of each period, \( \rho_0 \) fraction of employed workers from the previous period are separated from their jobs. Let \( n_{t-1} \) be the number workers who were employed in period \( t-1 \). The total number of job seekers in period \( t \) is then

\[
 u_t = 1 - (1 - \rho_0)n_{t-1}.
\]

Let \( v_t \) be the aggregate number of vacancies posted by the firms in the economy and \( m_t \) the number of matches formed. I follow the literature in assuming a Cobb-Douglas matching function:

\[
 m_t = m_0 u_t^\mu v_t^{1-\mu},
\]

where \( m_0 \) is the scale parameter and \( \mu \in (0, 1) \) is the match elasticity with respect to job seekers.

With all the ingredients in place, the law of motion for aggregate employment can be written as:

\[
 n_t = (1 - \rho_0)n_{t-1} + m_t.
\]

Note that given the quarterly timing, I allow a worker who is exogenously separated at the beginning of a period to—(1) join the pool of job seekers; (2) form a match with an employer; and (3) produce output—all within the same quarter. This implies the relevant unemployment statistics of the model that is comparable to data is

\[
 u_t^m = 1 - n_t,
\]

where \( u_t^m \) denotes measured unemployment. This corresponds to the number of workers who are not producing output at time \( t \).

Lastly, the job finding rate for a job seeker can be defined as:

\[
 s_t = \frac{m_t}{u_t};
\]

and likewise the vacancy fill rate:

\[
 q_t = \frac{m_t}{v_t}.
\]

### 3.2 Households

The economy consists of a continuum of households, each has an infinite number of identical members. I abstract from labor participation choice—every member of a household is either employed or is looking for work. Those who are employed receive wage income \( w_t h_t \), the hourly wage rate times the number of hours worked; and those who are not receive unemployment insurance \( b \) from the government. I assume that workers incur disutility from exerting effort once they are employed.

Each household member’s utility is additively separable in consumption and leisure, and there is
perfect risk-sharing among members of the household, yielding the same consumption for everyone in the household.

Let \( c_t \) denote consumption and \( h_t \) the intensive labor margin. Conditional on \( n_t \), the number of employed members, households’ objective function can be written as:

\[
E_t \sum_{s=0}^{\infty} \beta^s \left( \frac{1 - \gamma}{1 - \gamma} - \kappa h_t^{1+\phi} \right),
\]

(1)

where \( \beta \) is the discount factor; \( \gamma \) is the coefficient of relative risk aversion; \( \kappa \) is the scale parameter for disutility of work; and \( \frac{1}{\phi} \) is the intertemporal elasticity of substitution of leisure.

Households can save or borrow by investing in capital good \( k_t \) that depreciates at rate \( \delta \). A household receives rental income \( r_t \) for each unit of capital good it rents to firms in a competitive market. Households maximize their objective function (1) subject to a sequence of budget constraints:

\[
c_{t+s} + k_{t+s+1} - (1 - \delta)k_{t+s} \leq w_{t+s}h_{t+s}n_{t+s} + (1 - n_{t+s})b + r_{t+s}k_{t+s} + \Pi_{t+s} - T_{t+s},
\]

where \( \Pi_{t+s} \) is the dividend payments from firms and \( T_{t+s} \) is the lump-sum tax levied by the government to finance the unemployment insurance.

Households’ problem yields the standard Euler’s equation; let \( \lambda_t = u'(c_t) \), we have:

\[
\lambda_t = \beta E_t \lambda_{t+1}(r_{t+1} + 1 - \delta).
\]

For the purpose of wage and hour setting that will be discussed below, it is useful to write down the surplus of an employed worker to a household. Let \( U_t \) and \( W_t \) denote the value of an unemployed worker and employed worker, respectively. The value of an unemployment worker, in units of consumption good, is:

\[
U_t = b + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} [s_{t+1}W_{t+1} + (1 - s_{t+1})U_{t+1}];
\]

(2)

and the value of an employed worker is:

\[
W_t = w_t h_t - \frac{\kappa h_t^{1+\phi}}{1+\phi} \frac{\lambda_{t+1}}{\lambda_t} [(1 - \rho_0 + \rho_0 s_{t+1})W_{t+1} + \rho_0(1 - s_{t+1})U_{t+1}].
\]

(3)

Equation (2) says the value of an unemployed worker is the unemployment insurance she receives plus the continuation value weighted by the probability of finding a job in the next period. Equation (3) says the value of an employed worker is the wage payment she receives, less the disutility of effort, plus the continuation value weighted by the probability that she continues to have a job the next period.\(^{18}\) The surplus of an employed worker, \( M_t = W_t - U_t \) is then:

\[
M_t = w_t h_t - \frac{\kappa h_t^{1+\phi}}{1+\phi} \frac{\lambda_{t+1}}{\lambda_t} (1 - \rho_0)(1 - s_{t+1})M_{t+1}.
\]

(4)

\(^{18}\) To be more precise, with probability \( 1 - \rho_0 \) the worker will survive the exogenous separation shock; with probability \( \rho_0 s_{t+1} \) she will lose her job exogenously but will find a new job in period \( t + 1 \); and with probability \( 1 - [(1 - \rho_0) + \rho_0 s_{t+1}] = \rho_0(1 - s_{t+1}) \) she will be unemployed.
3.3 Firms

Let \( h_{j,t} \) be the hours each worker spent in production for firm \( j \) in period \( t \), \( n_{j,t} \) the number of workers, and \( k_{j,t} \) the units of capital employed by firm \( j \). I assume that a firm chooses the same hour for all of its workers; output is then:

\[
y_{j,t} = a_t k_{j,t}^\alpha \left( h_{j,t} n_{j,t} \right)^{1-\alpha}.
\] (5)

\( \alpha \in (0, 1) \) measures the diminishing returns on capital, and \( \vartheta \in (0, 1] \) is the additional diminishing return on the intensive margin. (Or, in the case of \( \vartheta = 1 \), constant returns.) \( \vartheta \) captures the notion that the worker becomes less and less effective the more and more effort is required of them.\(^{19}\)

The productivity process, \( a_t \), follows an autoregressive process:

\[
\log a_t = \rho_a \log a_{t-1} + \sigma_{a,t-1} \varepsilon_{a,t},
\] (6)

where \( \rho_a \) is the persistence parameter and the innovations \( \varepsilon_{a,t} \) are \( i.i.d. \ N(0, 1) \).

The standard deviation of the innovations above, \( \sigma_{a,t} \), itself follows an autoregressive process:

\[
\log \sigma_{a,t} = \rho_\sigma \log \sigma_{a,t-1} + (1 - \rho_\sigma) \log \bar{\sigma} + \eta_\sigma \varepsilon_{\sigma,t},
\] (7)

where \( \rho_\sigma \) is the persistence parameter, \( \bar{\sigma} \) is the non-stochastic mean of \( \sigma_t \), \( \eta_\sigma \) is the standard deviation of the innovations, and \( \varepsilon_{\sigma,t} \) is \( i.i.d. \ N(0, 1) \). Note the timing assumption is such that firms know in advance the distribution of next period’s innovations. This means that when an uncertainty shock hits today, agents realize that the innovation to productivity next period will come from a wider distribution. This represents the notion of uncertainty as firms make their decisions today.\(^{20}\)

Firms acquire capital goods for production from a competitive market at rental rate \( r_t \). Firms post vacancies to attract new workers. For firm \( j \) that begins period \( t \) with \( n_{j,t-1} \) units of labor and posts \( v_{j,t} \) vacancies, its employment law of motion is:

\[
n_{j,t} = (1 - \rho_0) n_{j,t-1} + q_t v_{j,t},
\]

where \( q_t \) is the economy-wide vacancy fill rate which is taken as given. The cost of employment adjustment is \( \frac{\kappa_v}{2} (q_t v_{j,t})^2 \).\(^{21}\)

Given that households own the firms, firms discount the future using households’ stochastic discount factor. Firm \( j \) chooses \( v_{j,t} \) and \( k_{j,t} \) to maximize the present value of its lifetime profits

\(^{19}\)Consider two scenarios. Scenario 1: 8 workers each working 10 hours; and scenario 2: 5 workers each working 16 hours. If \( \vartheta = 1 \) then the effective labor input, \( h_{j,t} n_{j,t} \), of the two scenarios are equivalent. \( \vartheta < 1 \) indicates that the workers in scenario 2 are less effective than the workers in scenario 1.


\(^{21}\)Quadratic adjustment cost is utilized here because it incentivizes firms to make gradual adjustments to their labor force. Relative to the standard per-vacancy adjustment cost, this assumptions improves the model’s ability to generate jobless recoveries, though it is not crucial. Merz and Yashiv (2007), Gertler, Sala, and Trigari (2008), and Galí and van Rens (2010) are a sample of recent literature that also utilize convex adjustment costs.
subject to employment law of motion. I assume firms and workers jointly determine wage and the
intensive margin through a process I will describe later. Firm $j$’s problem can be written as:

$$V_{j,t} = \max_{\{v_{j,t}, k_{j,t}\}} \left\{ y_{j,t} - w_{j,t} h_{j,t} n_{j,t} - r_t k_{j,t} - \frac{\kappa_v}{2} (q_t v_{j,t})^2 + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} V_{j,t+1} \right\},$$  \hspace{1cm} (8)

subject to

$$n_{j,t} = (1 - \rho_0) n_{j,t-1} + q_t v_{j,t}.$$  

Let $J_{j,t}$ be the Lagrangian multiplier for employment, the first order conditions for the firm’s
problem are:

$$v_{j,t} : \quad \kappa_v q_t v_{j,t} = J_{j,t} \hspace{1cm} (9)$$

$$k_{j,t} : \quad r_t = \alpha \frac{y_{j,t}}{k_{j,t}} \hspace{1cm} (10)$$

Condition (9) equates the marginal cost of hiring a new employee to the value of adding another
worker, $J_{j,t}$, which I will describe in more detail in the following section. Condition (10) is the
standard capital optimality condition.

Given that all the firms are identical, I will omit the $j$ subscript below.

3.4 Hours and Wage Setting

Due to labor market friction, employer-employee matches create a positive surplus to be shared
between the parties. In this model, firms and their workers jointly determine hours $h_t$ and wages
$w_t$.

3.4.1 Hours

In terms of hours, I assume that $h_t$ is set at the level such that the marginal product equals the
marginal disutility of the household. More specifically:

$$(1 - \alpha) \beta \frac{y_t}{h_t} = \frac{\kappa_h h_t}{\lambda_t} n_t,$$  \hspace{1cm} (11)

where the left hand side of the expression is the marginal product of labor at the intensive margin;
and the right hand side is the household’s marginal disutility of effort.

3.4.2 Wages

With the hours schedule specified above, the value of an additional worker to the firm can be
derived by taking the derivative of firm’s objective function (8) with respect to $n_t$ subject to (11)
and employment law of motion. It is:

$$J_t = (1 - \xi_F) (1 - \alpha) \frac{y_t}{n_t} - w_t h_t + (1 - \rho_0) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_{t+1},$$  \hspace{1cm} (12)
where \( \xi_F = \frac{\alpha}{1+\phi-\alpha(1-\alpha)} \) captures the endogenous effect of an additional worker on the hours choice—a firm recognizes when it hires an additional worker, it can reduce hours among all its existing workers. (See appendix A for the derivation of \( \xi_F \).) Expression (12) tells us that the value of a worker to the firm equals her marginal product, less the wage payment, plus the continuation value weighted by the probability the match survives the exogenous separation shock next period.

Before I proceed further, it is worth noting that workers would be willing to stay in a match with a firm as long as their surplus, \( M_t \), is positive; likewise, a firm is willing to stay in a relationship with a worker if \( J_t \) is positive. Using equations (4) and (12), this implies the lower bound of wage bill for a worker is

\[
\begin{align*}
  w^b_t h_t &= \frac{k_h h_t^{1+\phi}}{\lambda_t^{1+\phi}} + b + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} (1 - \rho_0) (1 - s_{t+1}) \left( w^b_{t+1} h_{t+1} - w_{t+1} h_{t+1} \right),
\end{align*}
\]

Similarly, the upper bound of the wages is

\[
\begin{align*}
  w^{ub}_t h_t &= (1 - \xi_F) (1 - \alpha) \frac{y_t}{n_t} + (1 - \rho_0) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left( w^{ub}_{t+1} h_{t+1} - w_{t+1} h_{t+1} \right).
\end{align*}
\]

In the context of this model, the Nash rent-sharing outcome that is standard in the literature is equivalent to:

\[
\begin{align*}
  w^N_t &= \eta w^{ub}_t + (1 - \eta) w^b_t,
\end{align*}
\]

where \( \eta \in [0, 1] \) is the worker’s share of the total surplus.

Shimer (2005) and Hall (2005) have argued that period-by-period Nash rent-sharing wage shown above is too volatile relative to the data, which results in a muted response of employment to productivity shocks. Hall (2005) further points out that any wage within the bargaining set, defined as any wage between \( w^b_t \) and \( w^{ub}_t \) should be considered a legitimate solution to the wage bargaining process between a firm and its employees. In order to allow the model to generate a more realistic employment response to productivity shocks, I adopt the following wage rule:

\[
\begin{align*}
  w_t &= \tau w_{t-1} + (1 - \tau) w^N_t,
\end{align*}
\]

where \( \tau \in [0, 1] \) indexes the degree of wage rigidity; I constrain \( w_t \in [w^b_t, w^{ub}_t] \). \(^{23}\)

3.5 Government and Resource Constraint

Government levies a lump-sum tax \( T_t \) from the households to finance unemployment insurance \((1-n_t)b\). Let \( x^* \) denote the non-stochastic steady-state value of variable \( x \), I assume the unemployment insurance \( b \) satisfies the condition:

\[
\begin{align*}
  b + \frac{k_h h^{1+\phi}}{\lambda^{1+\phi}} = \bar{b}(1-\alpha) \frac{y^*}{n^*},
\end{align*}
\]

\(^{22}\)One can derive the expression for \( w^b_t h_t \) and the expression for \( w^{ub}_t h_t \) by setting \( M_t = 0 \) and \( J_t = 0 \) and by noting \( M_t = w_t h_t - w_t^b h_t \), and \( J_t = w_t^{ub} h_t - w_t^b h_t \).

\(^{23}\)See Hall (2005) for a discussion of this particular adaptive wage determination process. This paper is one of many in the recent literature that departs from period-by-period Nash rent-sharing wage; see, for example, Gertler, Sala, and Trigari (2008), Shimer (2010), Blanchard and Galí (2010), and Galí and van Rens (2010).
that is, the unemployment insurance is set such that the opportunity cost of employment, $b$ and
the utility gained from supplying no effort, equals a constant fraction of the marginal product of
labor in the steady-state.

Lastly, to close the model, the resource constraint is

$$y_t = c_t + \frac{k_{t+1}}{2} (q_t v_t)^2 + k_{t+1} - (1 - \delta) k_t.$$ 

4 Calibration and Solution

This section details the calibration strategy used in this paper as well as the approach this paper
takes to examine the role uncertainty plays in jobless recoveries.

4.1 Calibration

I begin with the set of parameters that are directly comparable to the business cycle literature.
The capital share, $\alpha$ is calibrated to 0.33. The discount rate $\beta$ is set to 0.99. Capital depreciation
rate $\delta$ is 0.026, which corresponds to a 10% annual depreciation rate. I choose 0.5 to be the
intertemporal elasticity of substitution of hours. This is on the high end of the microeconomic
estimates, typically ranging between 0.1 to 0.5; this implies $\phi = 2$. Given that this model does
not have a non-convex adjustment cost, it is not surprising the curvature of the utility function
plays a role in agents’ response to uncertainty shocks; I set the coefficient of relative risk aversion,
$\gamma$, to 5. The disutility scale parameter $\kappa_h$ is set to normalize the intensive margin to 1 in the
non-stochastic steady-state.

The next set of parameters characterizes the labor market. I set $\bar{u}$, the non-stochastic steady-
state pool of job seekers, to 0.156—this yields 6.2% measured unemployment rate which is the
average U.S. unemployment rate between 1969Q1 and 2011Q2. I calibrate the non-stochastic
steady-state firm vacancy fill rate $q$ to 0.7, the same value used by den Haan, Ramey, and Watson
(2000), Cooley and Quadrini (1999), and Krause and Lubik (2007). The exogenous separation rate,
$\rho_0$ is set to 0.1. This is consistent with 0.034 monthly separation rate computed by Shimer (2005)
and is within the range of values used in the literature, ranging from 0.07 in Merz (1995) to 0.15
in Andolfatto (1996). I set both the elasticity of matches to unemployment, $\mu$, and the worker’s
bargaining power, $\eta$, to 0.5; they are within the range of values in the literature. While it appears
logical to consider decreasing returns to effort in effective labor input $h_t^\vartheta n_t$, the degree of decreasing

---

24 For a survey of the literature, see Card (1994).

25 While this is higher than the standard business cycle literature, Barsky, Juster, Kimball, and Shapiro (1997)
find that, based on survey responses to a hypothetical choice over the respondents’ lifetime income, when given a
50-50 lottery of—(1) double the respondents’ lifetime income, versus (2) reduce the respondents’ lifetime income by
one-fifth—approximately two-thirds of the respondents would reject the lottery and opt to keep their current income.
This decision corresponds to a lower bound of relative risk aversion of 3.8, suggesting that deviating from the usual
log-utility might not be entirely unreasonable.

26 The values of $\mu$ range from 0.4 in Merz (1995) to 0.72 in Shimer (2005). While my model is not directly comparable
to that of Gertler, Sala, and Trigari (2008), it is worth pointing out that they estimate a medium-sized monetary
DSGE model and find $\eta$ to be 0.907 under staggered wage contracting and 0.616 under flexible wage, highlighting a
wide range of plausible values for $\eta$. 

15
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.33</td>
<td>Capital share</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Discount rate</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.026</td>
<td>10% annual depreciation rate</td>
</tr>
<tr>
<td>$\phi$</td>
<td>2</td>
<td>Intertemporal elasticity of substitution of 0.5</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>5</td>
<td>Constant relative risk aversion</td>
</tr>
<tr>
<td>$\bar{u}$</td>
<td>0.156</td>
<td>6.2% measured unemployment, U.S average</td>
</tr>
<tr>
<td>$\bar{q}$</td>
<td>0.7</td>
<td>Non-stochastic steady-state vacancy fill rate</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>0.10</td>
<td>0.035 monthly separation rate</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.5</td>
<td>Elasticity of matches to unemployment</td>
</tr>
<tr>
<td>$\vartheta$</td>
<td>0.65</td>
<td>Decreasing returns to hours</td>
</tr>
<tr>
<td>$\bar{b}$</td>
<td>0.75</td>
<td>Replacement ratio</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.85</td>
<td>Wage index, weight of previous period’s wage</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>0.83</td>
<td>Productivity process persistence</td>
</tr>
<tr>
<td>$\rho_\sigma$</td>
<td>0.83</td>
<td>Uncertainty process persistence</td>
</tr>
<tr>
<td>$\bar{\sigma}$</td>
<td>0.05</td>
<td>Non-stochastic mean of $\sigma$</td>
</tr>
<tr>
<td>$\eta^\sigma$</td>
<td>$\log 1.35$</td>
<td>Standard deviation of uncertainty shocks</td>
</tr>
</tbody>
</table>

Table 1: Calibrated parameters. See Section 4.1 for details.

returns is unclear. In the benchmark case I set $\vartheta = 0.65$.\(^{27}\) I choose the “replacement ratio” $\bar{b}$ to be 0.75, it is between 0.4 in Shimer (2005) to 0.955 in Hagedorn and Manovskii (2008); it is comparable to 0.73 used in Mortensen and Nagypál (2007). The rigid wage parameter $\tau = 0.85$ is chosen so the baseline model without time-varying volatility reproduces approximately 80% of the volatility in employment.\(^{28}\)

The last set of parameters governs the laws of motion for the two driving forces of the model—the productivity process and the volatility process. They are $\rho_a$, $\rho_\sigma$, the two persistence parameters, $\bar{\sigma}$, the non-stochastic steady-state mean of $\sigma_t$, and $\eta^\sigma$, the standard deviation of uncertainty shocks. I calibrate these parameters as part of the model evaluation procedure I describe below. As a preview, they are $\rho_a = 0.83$, $\rho_\sigma = 0.83$, $\bar{\sigma} = 0.05$; and $\eta^\sigma = \log 1.35$. $\eta^\sigma = \log 1.35$ implies that a one standard deviation shock to uncertainty in the non-stochastic steady-state raises $\sigma$ from $\bar{\sigma}$ to $(1.35)^1 \bar{\sigma}$, and a two standard deviation shock is $(1.35)^2 \bar{\sigma} \approx 1.82 \bar{\sigma}$, and so on. Table 1 summarizes the calibrated parameters.

### 4.2 Model Evaluation Strategy

The model is solved by third-order perturbation methods; third-order solutions allow me to simulate the model with second-order perturbations. Recall that this model has two stochastic processes, \(^{(6)}\)

\(^{27}\)I explore both a smaller and a larger $\vartheta$—0.3 and 1; it turns out it makes no qualitative impact and very little quantitative difference. See appendix B for more details.

\(^{28}\)As has been shown in the literature, the value of $\bar{b}$ plays an important role in employment dynamics—the higher the $\bar{b}$, the less responsive the wage is to productivity shocks, and the more volatile employment is in response to these shocks. The same logic extends to $\tau$ in the rigid wage models in that the stickier the wage, the less responsive the wage is to productivity shocks. I explore different values of $\bar{b}$ and $\tau$ in section B.
and (7). The solution yields policy functions as functions of these two stochastic processes.\footnote{To be technically correct, the policy functions also depend on the initial condition of the endogenous variables. However, given that I always use the endogenous variables’ steady-state values as the initial condition, I suppress the policy functions’ dependence on them to save notation.} Let \( a^T \) denote a sequence of productivity \( \{a_t\}_{t=0}^T \) and \( \sigma^T \) a sequence of time-varying volatility \( \{\sigma_t\}_{t=0}^T \). I define \( y\left(a^T, \sigma^T\right) \) and \( n\left(a^T, \sigma^T\right) \) as the model output and employment driven by \((a^T, \sigma^T)\). Let \( y^d \) denote the HP de-trended quarterly U.S. real GDP, \( n^d \) the HP de-trended quarterly U.S. non-farm business payroll, both with a smoothing parameter of 1,600, and \( T \) denote the number of observations. In order to evaluate the model, I carry out the following exercise:

**Step 1:** Initialize output and employment (along with all the other endogenous variables) at their steady-state values. Initialize \( a_0 = 1 \) and \( \sigma_0 = \bar{\sigma} \).

**Step 2:** For \( t = 1 \), solve the minimization problem:

\[
\min_{\{a_1, \sigma_1\}} \left[y_d^t - y(a^1, \sigma^1)\right]^2 + \left[n_d^t - n(a^1, \sigma^1)\right]^2.
\] (14)

That is, the algorithm solves for \( a_1 \) and \( \sigma_1 \) such that the sum of square differences between the model and the data output and employment is minimized.

**Step 3:** If \( t = T \), we are done; else take \((a^{t-1}, \sigma^{t-1})\) as given, repeat step 2.

This procedure solves, period-by-period, the productivity and volatility required to match data on output and employment; it yields a productivity series and a volatility series that together allow my model to match the data output and employment. From now on I refer to these model-implied processes \( \tilde{a}^T \) and \( \tilde{\sigma}^T \).

Not surprisingly, the resulting productivity and volatility processes depend on the parameter values of \( \rho_a, \rho_\sigma, \bar{\sigma}, \) and \( \eta^\sigma \). In order to calibrate these parameters I carry out a “fixed point” algorithm in which I begin with a set of initial parameters.\footnote{The initial values I had chosen were \( \rho_a = 0.859^{1/4}, \bar{\sigma} = 6.71\%, \eta^\sigma = \log 1.93 \)—these values were calibrated in 7. As a starting point, I picked \( \rho_\sigma = 0.859^{1/4} \) to match the persistence of \( a \).} Using the initial parameter values, I follow the algorithm described above to find \( \tilde{a}^T \) and \( \tilde{\sigma}^T \) to match data on output and employment. I then estimate equations (6) and (7) using \( \tilde{a}^T \) and \( \tilde{\sigma}^T \). After updating the parameters with the new estimates, I repeat the process until the parameters are sufficiently close between iterations.\footnote{In the last iteration, \( \bar{\sigma} \) was calibrated to 0.05 and the resulting point estimate was 0.048; \( \rho_a \) was calibrated to 0.83 and the point estimate was 0.829; \( \rho_\sigma \) was calibrated to 0.83 and the point estimate was 0.825. \( \eta^\sigma \) has no effects on the dynamics of the model; \( \eta^\sigma = 1.35 \) is chosen to ensure the standard deviation of \( \varepsilon_{a,t} \) is close to 1—it was 0.903. The shortcoming of this model is that \( \bar{\sigma} = 0.05 \) is larger than what is typically estimated as the standard deviation of the total productivity process (usually around 0.01); with \( \bar{\sigma} \) at 0.05, the standard deviation of the innovations, \( \varepsilon_{a,t} \), is approximately 0.20 which falls short of the assumption that \( \varepsilon_{a,t} \sim N(0, 1) \). One plausible explanation, as mentioned earlier in the paper, is that this procedure naively assumes that \( \tilde{a}^T \) and \( \tilde{\sigma}^T \) are the only processes driving output and employment. As a result, the model is likely to require counterfactually large uncertainty shocks to match the data.}

Given that the exercise is designed to allow my model to match data on output and employment by varying volatility, it is important to ask two questions of the constructed series:

**Question 1.** Does the model fail to replicate jobless recoveries once I counterfactually remove uncertainty shocks?
Question 2. Does the volatility series appear to be a reasonable measure of uncertainty? I will address these questions in the following section.

5 Results

This section provides the results from the numerical exercise described in the previous section.

5.1 Impulse Response Functions

Figure 4 plots the impulse response function to a two standard deviation shock to uncertainty from the non-stochastic steady-state. The $y$-axis plots percentage deviation from steady-state; the $x$-axis is in quarters. When an uncertainty shock hits the economy, we observe an immediate drop in vacancies. At the same time, firms require an increase in hours to partially make up for the lost workforce. This confirms our intuition that costly labor adjustment along the extensive margin leads firms to reduce hiring and substitute the extensive margin with effort. Due to continuing exogenous separations, a drop in vacancies leads to a decrease in employment and an increase in unemployment.

![Impulse response functions](image)

Figure 4: Impulse response of a two standard deviation shock to the time-varying volatility, $\sigma_t$. $y$-axis is percentage deviation from steady-state. $x$-axis is quarters.

This corresponds to an increase of $\sigma = 0.05$ to $\sigma = 0.09$. 

---

[^32]: This corresponds to an increase of $\sigma = 0.05$ to $\sigma = 0.09$. 

5.2 Computational Results

The rest of this section is organized as follows. First I will show that, without time-varying volatility, the model employment largely co-moves with output and has difficulty matching the recent episodes of jobless recoveries.\footnote{The standard deviation of productivity shocks $\sigma_t$ is held constant at $\bar{\sigma}$ for all $t$.} This is not surprising given the puzzling nature of jobless recoveries. Second, I will show the results with time-varying volatility and address the two questions I posed earlier. Based on these results, I conclude that uncertainty does indeed play an integral role in recent episodes of jobless recoveries.

Figure 5: Model and data output and employment. The $y$-axes are deviation from HP trend (smoothing parameter 1,600). The top panel plots output and the bottom panel employment. The red line is data and the pink line (with $\times$s) is the model without time-varying volatility.

Figure 5 shows model and data output and employment when productivity is the only stochastic process driving the model. The pink lines with $\times$s are the model output and employment with time-varying volatility turned off. ($\sigma_t$ is held constant at $\bar{\sigma}$.) The red lines are the HP de-trended (with smoothing parameter 1,600) U.S. GDP and U.S. non-farm payroll. We can see that, with productivity alone, the model output and employment strongly co-move with each other; the correlation coefficient of the two variables is 0.977 when it is 0.803 in the data. As a consequence of that strong correlation, we do not observe jobless recoveries. To summarize, we do not observe jobless recoveries when productivity is the only driving force of the model.
Next I will show the results from the full model, which refers to the simulation utilizing both productivity and uncertainty shocks to match data.

![Graph showing model and data output and employment.](image)

**Figure 6**: Model and data output and employment. The $y$-axes are deviation from HP trend (smoothing parameter 1,600). The top panel plots output and the bottom panel employment. The red line is data, the pink line (with $\times$s) is the model without time-varying volatility, and the blue line (with circles) is the simulation with both productivity and uncertainty shocks.

Each panel of Figure 6 shows three series. Series (1) is the blue line (with circles); it represents the simulation with both productivity and uncertainty shocks. Series (2) is the pink line (with $\times$s); it shows the counterfactual output and employment with the same productivity process as series (1), but instead of allowing time-varying volatility as series (1), $\sigma_t$ is held constant at $\bar{\sigma}$ for all $t$. Series (3) is the U.S. real GDP or non-farm payroll, HP de-trended with a smoothing parameter 1,600.

Several observations can be made of Figure 6:

1. Given the minimal impact of uncertainty shocks on output, the model output is largely driven by the productivity process.\(^{34}\) Therefore, using actual output as a frame of reference, by comparing the dynamics of output in Figures 5 and 6, one can conclude that productivity is the main driving force of this model and that adding uncertainty does not drastically alter the dynamics of the model.

\(^{34}\)This can be seen in Figure 4 where hours and employment move in different directions but with comparable magnitudes; as a result, they largely offset each other in their impacts on output.
2. Time-varying volatility allows the model to fully replicate the evolution of actual output and employment. This is shown by series (1) and (3) overlapping each other in Figure 6.

3. As seen in Figure 5, employment co-moves with output in the absence of uncertainty shocks and thus fails to replicate jobless recoveries. This observation answers question 1 in the affirmative.

Figure 7: Model-implied volatility and corporate profit forecast disagreement. The blue line (with circles) is the model-implied volatility; the red line is the one-quarter-ahead corporate profit forecast disagreement.

To address question 2, I turn to Figure 7. Figure 7 shows the model-implied volatility in blue (with circles) on the left y-axis and the corporate profit forecast disagreement (as shown in Figure 3) in red on the right y-axis. The correlation coefficient of the two series is 0.233. The model-implied volatility generally tracks the movement of corporate profit forecast disagreement, and most importantly, periods of heightened uncertainty following the three contractions are captured here. The model-implied volatility shows three sets of peaks, corresponding to each of the three jobless recoveries. The first peak occurred in 1992Q4, with the model suggesting increases in uncertainty throughout 1992. On the footsteps of the early 1990s recession, this period of high uncertainty corresponds to the 1992 presidential election in which the economy and taxes were central themes of the campaign. The second set of peaks occurred between 2003Q3 and 2005Q1. This period followed the invasion of Iraq in 2003 and was characterized by steadily rising oil prices
which could cast doubt on the strength of the recovery following the early 2000s recession. The last set of peaks in model-implied uncertainty occurred between 2010Q1 and 2010Q3. This comes at the heel of a contentious health care reform debate, which many business managers may perceive as a potential burden; add to that were uncertainties surrounding whether the Bush tax cuts would be allowed to expire or extended. All of these could have contributed to the lackluster recovery in employment.

To summarize, given that I can account for both the general shape and the specific peaks in uncertainty, I conclude question 2 in the affirmative—the model-implied volatility does indeed appear to be a reasonable measure of uncertainty.

Since the model relies on the intensive margin as a substitute for the extensive margin during episodes of jobless recoveries, I compare the model aggregate hours ($h_t n_t$) with the actual aggregate hours (non-farm business hours of all persons) to verify the validity of the model. This is shown in Figure 8. It can be seen that while the model hours is less volatile than that of the data, the two shares similar dynamics. In fact, the correlation coefficient of aggregate hours of the full model (blue line with circles) and the data (red line) is 0.888. This suggests that the model does not rely on unrealistic intensive margins to generate jobless recoveries—this finding further strengthens the result of this paper.

![Figure 8: Model and data aggregate hours.](image)

The $y$-axis is deviation from HP trend (smoothing parameter 1,600). The red line is data; the pink line (with $\times$s) is the model without time-varying volatility; and the blue line (with circles) includes both productivity and uncertainty shocks.
To establish the robustness of the results, I explore several alternative specifications of the model parameters. See appendix B for more details.

6 Conclusion

This paper finds that recent downturns have the following characteristics that set them apart from the earlier recessions: (1) High uncertainty about economy-wide corporate profits accompanies the turning points of output; and (2) The troughs of employment coincides with the peaks of uncertainty following a recession. Together, these two characteristics suggest that high uncertainty could hold back employment recovery despite positive output growth; and that once uncertainty begins to fall, employment begins to climb following a recession. This is consistent with our intuition that managers act more cautiously when they are unsure of their future prospects.

To account for these findings, I develop a dynamic stochastic general equilibrium model that features search-and-matching frictions in the labor market and an intensive labor margin, where the model is driven by productivity and time-varying volatility shocks. When an uncertainty shock hits the economy, firms do not want to make a costly mistake so they substitute away from the extensive margin toward the intensive margin, leading to jobless recoveries. I calibrate the model and show that the model is quantitatively capable of replicating episodes of jobless recoveries with reasonable uncertainty shocks. Without uncertainty shocks, the model fails to generate jobless recoveries. The failure of model to replicate jobless recoveries when uncertainty shocks are moved allows me to conclude that uncertainty does play an integral role in recent episodes of jobless recoveries.

The parsimonious nature of this model makes it uniquely suited to be adopted in a wide class of macroeconomic models to help us better understand the labor market. Indeed, it can be used as a foundation to address how financial frictions, policy shocks, and policy uncertainty shocks play a role in jobless recoveries. From a different perspective, given highly publicized nature of jobless recoveries, it will be interesting to understand how prolonged joblessness might contribute to agents’ perception of uncertainty. These topics are left for future research.
A Marginal Product of Labor

For convenience, the hours condition is rewritten here:

\[(1 - \alpha) y_t \frac{h_t}{n_t} = \frac{\kappa_h h_t^\phi}{\lambda_t} n_t.\]

Substitute the production function (5) for \(y_t\) and rearrange, we get:

\[h_t^{1 + \frac{\phi - \vartheta(1 - \alpha)}{\alpha}} = (1 - \alpha) \frac{\lambda_t}{\kappa_h} \vartheta a_t k_t^\alpha n_t^{-\alpha}.\]

Implicitly differentiate:

\[\frac{1 + \phi - \vartheta(1 - \alpha)}{1 + \phi - \vartheta(1 - \alpha)} \frac{\partial h_t}{\partial n_t} = -\alpha(1 - \alpha) \vartheta \frac{\lambda_t}{\kappa_h} a_t k_t^\alpha n_t^{-\alpha - 1} \frac{\partial n_t}{\partial n_t}.\]

Rearrange, substitute the production function back, and use the hours condition we get:

\[\frac{\partial h_t}{\partial n_t} = \frac{-\alpha}{1 + \phi - \vartheta(1 - \alpha)} \frac{h_t}{n_t}.\]

This expression gives us the reduction in hours when an additional worker joins the firm.

The marginal value of labor is

\[\frac{dy_t}{dn_t} = \frac{\partial y_t}{\partial n_t} + \frac{\partial y_t}{\partial h_t} \frac{\partial h_t}{\partial n_t} = (1 - \alpha) \frac{y_t}{n_t} \left[1 + \vartheta \frac{n_t}{h_t} \frac{\partial h_t}{\partial n_t}\right] = (1 - \alpha) \frac{y_t}{n_t} \left[1 + \frac{\alpha \vartheta}{1 + \phi - \vartheta(1 - \alpha)} \right].\]

Defining \(\xi_F = \frac{\alpha \vartheta}{1 + \phi - \vartheta(1 - \alpha)}\) gives us equation (12).

B Robustness

In this section I explore different data filtering scheme, different decreasing returns to hours \(\vartheta\), and different combinations of replacement ratio \(\bar{b}\) and rigid wage index \(\tau\).

B.1 Different Filtering Scheme

The model evaluation procedure is carried out with data filtered using HP 10^5 (as in Shimer (2005)) and band-pass filter with smoothing parameter 6 and 32 (typical for quarterly data, see Christiano and Fitzgerald (2003)). Figures 9 and 10 are analogous to Figure 6 and show the behaviors of simulated output and employment with and without uncertainty shocks. The full model is able to match both output and employment regardless of the filtering scheme. The counterfactual employment series without uncertainty fails to generate jobless recoveries as in the case of HP 1,600. Figure 11 shows the uncertainty processes under these three filtering schemes. The volatility series under HP 1,600 is the blue line (with circles); HP 10^5 is pink (with ×s); and band-pass filter...
is red (with diamonds). As can be seen in Figure 11, the model-implied uncertainty does not change dramatically when different filtering scheme is used. The correlation coefficient of the blue line and pink line is 0.934; the correlation coefficient of blue line and red line is 0.708; and the correlation coefficient of the pink and the red line is 0.667.

![Figure 9: Model and data output and employment, HP filter 10^5](image)

**B.2 Decreasing Returns to Hours**

Since different values of ϑ does not alter the fact that the model cannot replicate jobless recoveries without uncertainty shocks, I suppress figures of output and employment in the interest of space. However, when one looks at Figure 12 which shows the model-implied volatility series under different values of ϑ, it is quite clear that different decreasing returns to hours have very little impact on the results of the model. The smallest correlation coefficient among the 3 series is 0.958 between ϑ = 1 and ϑ = 0.3.

**B.3 Replacement Ratio and Rigid Wage Index**

Here I explore 2 alternative specifications: (1) \( \bar{b} = 0.75 \) and \( \tau = 0.8 \); and (2) \( \bar{b} = 0.83 \) and \( \tau = 0.85 \).\(^{35}\) The standard deviation of employment under alternative specification (1) is approximately 70% of the data volatility when we turn time-varying volatility off and force the model to match output only.

\(^{35}\) Recall the benchmark values are \( \bar{b} = 0.75 \) and \( \tau = 0.85 \). With these values, employment is approximately 80% of the data volatility when we turn time-varying volatility off and force the model to match output only.
Figure 10: Model and data output and employment, band-pass filter, 6 and 32

the data; it is 90% under alternative specification (2).

Figure 13 shows the intuitive result that the more of the employment dynamics I leave for the uncertainty shocks, the larger the uncertainty shocks are required to match the data. This can be seen in alternative specification (1) whose employment is approximately 70% as volatile as the data requires the largest uncertainty shocks; alternative specification (2), on the other hand, requires the smallest uncertainty shocks. It should be noted, however, that the correlation among the volatility processes remain high. The correlation coefficients between the benchmark and alternative 1 and 2 are 0.985 and 0.941 respectively; the correlation coefficient of alternative 1 and 2 is 0.876.
Figure 11: Uncertainty under different filtering schemes, including HP 1,600, HP $10^5$, and band-pass filter 6 and 32.
Figure 12: Uncertainty under different decreasing returns to hours, including $\vartheta = 0.3$, $\vartheta = 0.65$ (benchmark case), and $\vartheta = 1$.  

Model-implied volatility under different decreasing returns parameter

- $\vartheta = 0.65$
- $\vartheta = 1$
- $\vartheta = 0.3$
Figure 13: Uncertainty under different specifications of $\bar{b}$ and $\tau$. 

Model-implied volatility under different replacement ratio and rigid wage index

Benchmark
Alternative 1
Alternative 2
References


