1. Suppose \((a, b) = 1, a \mid c, \text{ and } b \mid c\). Show that \(ab \mid c\).

Because \((a, b) = 1\) we know that there exist \(x\) and \(y\) such that \(ax + by = 1\). Thus, \(acx + bcy = c\). Now, \(b \mid c\) so it follows that \(ab \mid ac\) and \(a \mid c\) so it follows that \(ab \mid bc\), but this implies that \(ab\) divides all linear combinations of \(ac\) and \(bc\) so in particular it must divide \(acx + bcy = c\).

Alternatively, from the hypotheses we know that there exists some integer \(m\) such that \(c = ma\). Given that \(b \mid ma\) and \((b, a) = 1\) it follows from class that \(b \mid m\). In particular there exists some integer \(n\) so that \(m = bn\). But now \(c = am = abn\) and thus \(ab \mid c\).

5. Prove that if \((a, m) = (b, m) = 1\) then \((ab, m) = 1\).

Assume not. Then there must exist some prime \(p\) such that \(p \mid (ab, m)\). In particular, this implies that \(p \mid m\) and \(p \mid ab\). Because \(p\) is prime the latter fact implies that either \(p \mid a\) or \(p \mid b\) (or both). But this in turn implies that either \(p \mid (a, m)\) or \(p \mid (b, m)\) contradicting our hypothesis.

6. The correct answer to part d was that \(\nu(p_1^{e_1} \ldots p_r^{e_r}) = (e_1 + 1)(e_2 + 1) \ldots (e_r + 1)\). The answers to parts a-c are just special cases of this formula. This formula can be proven in several different ways.