Power Rating System

Introduction

The Power Rating System is a ranking system that takes ranks teams based upon their number of wins and strength of schedule. The tougher scheduled opponents a team plays can greatly impact their final ratings if they are able to win them all. This means that if Team A beats 5 teams that have each won 10 games, but loses 3 games, and Team B beats 8 teams that have won 4 games each, then Team A would be ranked higher because they have beaten more difficult teams. To find a teams rating you make a matrix of all teams being rated, and then add the same matrix multiplied by the same matrix with a given weight to the previous matrix. Then you add up the row corresponding to the team.

An Example

Lets look at an example of six teams from the 2012 Division III college football season: Johns Hopkins, Franklin & Marshall, Gettysburg, Dickinson, McDaniel, and Juniata with a weight of 1 and a power of 1. Consider following chart of these teams’ (in the rows) wins and losses against teams (in the columns):

<table>
<thead>
<tr>
<th>Teams</th>
<th>Johns Hopkins</th>
<th>F &amp; M</th>
<th>Gettysburg</th>
<th>Dickinson</th>
<th>McDaniel</th>
<th>Juniata</th>
</tr>
</thead>
<tbody>
<tr>
<td>Johns Hopkins</td>
<td>-</td>
<td>Loss, 12 - 14</td>
<td>-</td>
<td>Win 49-0</td>
<td>Win 49-7</td>
<td>Win 40-20</td>
</tr>
<tr>
<td>F &amp; M</td>
<td>Win, 14 - 12</td>
<td>-</td>
<td>Loss, 31-38</td>
<td>Win 36-28</td>
<td>-</td>
<td>Win 45-38</td>
</tr>
<tr>
<td>Gettysburg</td>
<td>-</td>
<td>Win, 38-31</td>
<td>-</td>
<td>Loss 23-13</td>
<td>Win 35-3</td>
<td>Win 28-7</td>
</tr>
<tr>
<td>Dickinson</td>
<td>Loss 0-49</td>
<td>Loss 28-36</td>
<td>Win 13-23</td>
<td>-</td>
<td>Win 38-31</td>
<td>-</td>
</tr>
<tr>
<td>McDaniel</td>
<td>Loss 7-49</td>
<td>-</td>
<td>Loss 3-35</td>
<td>Loss 31-38</td>
<td>-</td>
<td>Loss 7-24</td>
</tr>
<tr>
<td>Juniata</td>
<td>Loss 20-40</td>
<td>Loss 38-45</td>
<td>Loss 7-28</td>
<td>-</td>
<td>Win 24-7</td>
<td>-</td>
</tr>
</tbody>
</table>

Here is what a matrix would look like in the same order as the table.
\[
A = \begin{bmatrix}
0 & 0 & 0 & 1 & 1 & 1 \\
1 & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\]

The original matrix \( A \), starts with all zeros, then \( A \) to the first power is added to the original matrix. There are several ways to rank these teams. Look at the following one:

1. 3.0000 Franklin and Marshall
2. 3.0000 Johns Hopkins
3. 3.0000 Gettysburg
4. 2.0000 Dickinson
5. 1.0000 Juniata
6. 0.0000 McDaniel

This ranking has Franklin and Marshall ahead of Johns Hopkins despite their having the same records.
This team has:

1. Johns Hopkins
2. Franklin and Marshall
3. Dickinson
4. Gettysburg
5. Juniata

The same could be done with Gettysburg being ranked first. Now let’s assume the matrix is raised to the second power with the first matrix being weighted 1 and the second matrix weighted .5. By adding $A \times .5 \times A$ the new rankings look something like this:

1. 6.0000 Franklin and Marshall
2. 5.0000 Gettysburg
3. 4.5000 Johns Hopkins
4. 3.5000 Dickinson
5. 1.0000 Juniata
6. 0.0000 McDaniel

As you can see, Franklin and Marshall is now the best team because they had the highest number of quality wins. What happened was Franklin and Marshall won 3 games and the weight of the first matrix is 1, so they get 3 points from the first matrix, then they add on
half of all the wins from the opponents they beat. They receive half because the weight of the second matrix was established at .5.

The Math

When you have a matrix $A$, and you want to find out the rating score for team $i$, all you do is add up all the columns in row $i$. However, to find a more precise ranking that takes into account the strength of schedule you multiply matrix $A$ by matrix $A$. For matrix $A$:

$$
A = \begin{bmatrix}
a_{11} & a_{12} & a_{13} & \ldots & a_{1n} \\
& \ddots & & & \\
& & \ddots & & \\
& & & \ddots & \\
an_{1} & an_{2} & an_{3} & \ldots & ann
\end{bmatrix}
$$

Multiplying $A \times A$ gives

$$
B = \begin{bmatrix}
(a_{11} \times a_{11} + a_{12} \times a_{21} + a_{13} \times a_{31} + \ldots + a_{1n} \times an_{1}) & \ldots & (a_{11} \times an_{1} + a_{12} \times an_{2} + a_{13} \times an_{3} + \ldots + a_{1n} \times ann) \\
& \ddots & & & \\
& & \ddots & & \\
& & & \ddots & \\
& & & & \ddots & \\
& & & & & (an_{1} \times a_{1n} + an_{2} \times a_{2n} + an_{3} \times a_{3n} + \ldots + ann \times ann)
\end{bmatrix}
$$

By multiplying each row by each column, the $i$ row gets how many wins each team in column $j$ has multiplied by the weight and puts it in row $i$ column $j$ position. Let $b_{ij} = a_{ij} \times a_{ji}$ in matrix $B$. As shown above, $b_{ij} = \text{row } a_{ij} \times \text{column } a_{ji}$, which means that wherever $a_{ij} = 1$ or $1 \times$ the weight, and if $a_{ij} = 1$, then team $i$ beat team $j$, so it gets added into $b_{ij}$. To get the rating you then take add the rows to the existing matrix. Repeat this process until all the weights are done. For each power, that is how many opponents’ wins are added to the
ratings. The first power takes into account the one team’s wins, the second power takes into account all of its opponent’s wins, the third power adds the opponents’ opponents’ wins, and so on.

**Advantages and Disadvantages**

The advantages of this method is that it takes into account how strong of opponents a team is able to beat. Playing against teams with better records benefits a team because it gives them a chance to rank higher given that the power is greater than two. This method rewards teams that is able to beat other good teams, not just rewarding teams that win all their games, but play an easy schedule, once again assuming the power is greater than two. A disadvantage of this method is that it is much more effective if all teams play the same number of games. For instance, there is a league in Division III football that only plays eight games, as opposed to other teams that play ten games, thus giving teams the opportunity to win more games than the one league; more games means other teams also have an opportunity to gather more weighted opponents’ wins. Another disadvantage is that this method does not take score for close games into account, which could help teams that lose a lot of games by close scores.

**Possible Adjustments**

A few possible fixes to this method would be to increase the number of powers used and to change the weights given to each power of matrices. Adding more powers further the impact of the strength of schedule. Changing the weights can make the opponents’ wins seem stronger or weaker depending on the user’s choice. Another adjustment that could be made is to find the least amount of games a team played and take random games from each team that played more than that until their game totals match the least total number of amount games played by a team.
Conclusion

This method works well because it takes into account how often a team can win a game, and how well a team can win against stronger opponents. The stronger a schedule with a lot of wins can accurately reflect who is truly a contender and who is a pretender.