Reflections in a polished tube

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When one of us (E.B.M.) dislodged a metal tube from an electric door chime recently, she inadvertently introduced her father to an attractive and instructive optical phenomenon. Looking down the highly polished inner surface of the cylinder we could see a spot surrounded by a series of bright concentric rings. The pattern looked much like the display of fringes produced by a Fabry-Perot or Michelson interferometer, except that the rings were more evenly spaced instead of crowding together strongly near the edge of the field of view.

The phenomenon is easily explained by simple ray-optics as Fig. 1 illustrates. The observed central spot is the small hole at the opposite end of the tube, from which the chime is normally suspended. Concentric rings are formed by rays that pass through this hole and reflect one or more times from the polished sides. An examination of the geometry of the rays indicates that if D is the diameter of the tube and L is its length, rays visible to the observer at a point P in the open end of the tube will be inclined to its long axis by an angle θ_n given by:

$$\tan \theta_n = \frac{D}{L} n \qquad (1)$$

where n is the number of reflections the ray undergoes in passing through the tube. If the right-hand term is much less than 1, i.e., for relatively thin tubes and relatively small numbers of reflections, we can write:

$$\theta_n = \frac{D}{L} n \qquad (2)$$

For a typical door chime, with D of 1 in. and L of 36 in., we expect to see concentric circles with angular diameters of 3.2°, 6.4°, 9.6°, etc., corresponding to rays reflected 1, 2, and 3 times, respectively. The higher order rings should obey Eq. (1) more closely, crowding together perceptibly at larger diameters. We noticed this for rings of order 8 or so in our chime.

The number of rings visible to an observer should depend not only on the intensity of illumination at the hole, but also on the reflectivity of the sides of the tube. Since each succeeding ring represents rays which have undergone one more internal reflection than the preceeding one, the intensity of a ring which has undergone N more reflections than an inner one is decreased by a factor R^{-N}, where R is the fraction of incident light reflected by the inner tube walls. We could clearly distinguish 10 concentric rings when our chime was directly illuminated by a 100-W bulb.

This striking phenomenon is more than a mere novelty, a kaleidoscope with effectively an infinite number of mirror planes. There are similarities here to the propagation of light in an optical fiber, though that involves refraction, rather than reflection, at the walls and though fibers are thinner, longer, and can be bent. It is illustrative of the light-scattering effects one encounters in the design of light baffles for telescopes and various types of collimating devices. An analysis of the phenomenon, presented as an unexplained puzzle, might be an interesting problem for an introductory physics laboratory, since it is straightforward and it involves methods of reasoning that also apply to the description of standing waves on strings and in organ pipes. In short, there is something here to pique the native curiosity of a three-year-old and to appeal to the more educated sensibilities of high-school and college students.

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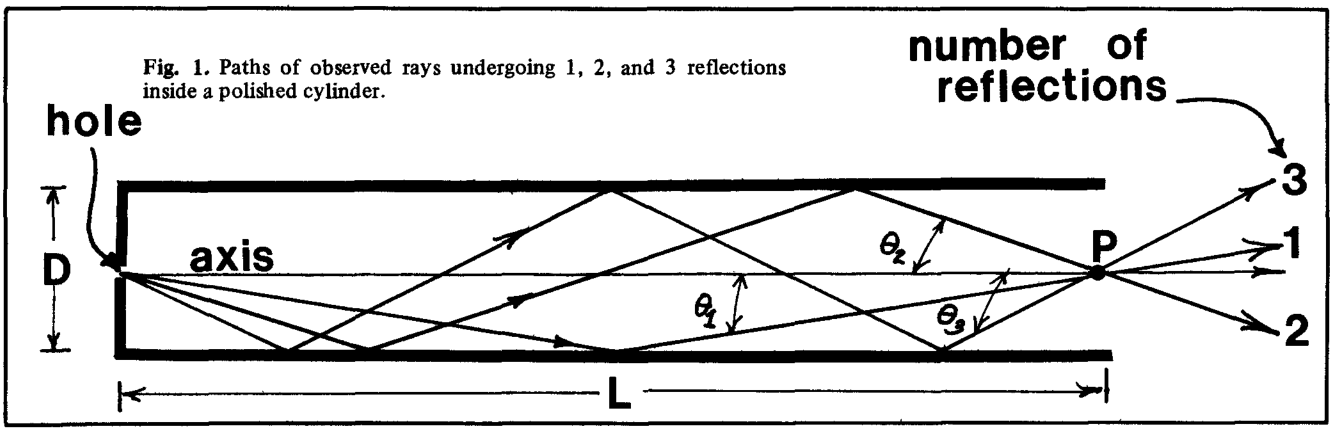

Fig. 1. Paths of observed rays undergoing 1, 2, and 3 reflections inside a polished cylinder.