The measurement of astronomical parallaxes with CCD imaging cameras on small telescopes

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Small telescopes equipped with charge-coupled device (CCD) imaging cameras are well suited to introductory laboratory exercises in positional astronomy (astrometry). An elegant example is the determination of the parallax of extraterrestrial objects, such as asteroids. For laboratory exercises suitable for introductory students, the astronomical hardware needs are relatively modest, and, under the best circumstances, the analysis requires little more than arithmetic and a microcomputer with image display capabilities. Results from the first such coordinated parallax observations of asteroids ever made are presented. In addition, procedures for several related experiments, involving single-site observations and/or parallaxes of earth-orbiting artificial satellites, are outlined.

I. INTRODUCTION

In growing numbers, undergraduate educational institutions are equipping their instructional and research telescopes with imaging charge-coupled device[1] (CCD) cameras, thereby providing their students with experience with modern astronomical detectors as well as making it possible for them to obtain images of faint and/or low-surface brightness objects. The linearity of the CCD response, high quantum efficiency, and the fact that the images are immediately available in digital form on a rectangular grid makes it easier to analyze the images quantitatively than it ever has been with photographic plates.

Currently, the most popular astronomical applications of these new detector systems at the undergraduate level, for both educational and scientific purposes, involve photometry of point sources, especially of stars. Such experiments are actually quite demanding of the students, as they require observations to be made over a period of many hours or days, followed by careful and computation-intensive image processing in order to transform pixel values to stellar intensities. In this paper we would like to describe another application of these powerful detectors which takes advantage of their sensitivity and metric stability, but which is still accessible to introductory-level students: the determination of the distances to celestial objects, especially asteroids, by measurement of their parallaxes.

In most introductory astronomy classes, students learn the importance of the observation of stellar parallax, both for the development of cosmological theories from ancient Greece up through the Renaissance, and in modern astronomy as one of the key observational techniques used in the determination of the galactic distance scale.[2] Nonetheless, with the possible exceptions of simultaneous photography of the moon in the vicinity of a bright planet or star, or of a relatively bright artificial satellite, from two or more reasonably widely separated geographical locations, we are unaware of any exercise dealing with the parallax of celestial objects that may have been performed regularly by introductory students using observational material which they gathered themselves.

The proximity of asteroids means that detectable parallaxes can be observed by telescopes separated by only a few hundred kilometers. With the availability of CCD camera systems, small telescopes are now powerful enough to detect hundreds or thousands of asteroids; furthermore, each image of a field containing an asteroid will simultaneously record several stars for use as positional references. A plentiful supply of asteroids (some of which pass quite close to the earth) makes it relatively easy to plan an asteroid parallax project.

The data reduction can be performed on microcomputers, and the image processing part of the analysis, such as compensating for dark current and pixel-to-pixel variations in sensitivity, which sometimes makes the analysis of CCD

observations tedious for introductory students, will typically be far less important than for photometric projects, and can probably be ignored altogether (or saved for the next lab!). Students with little foundation in modern physics will still be able to appreciate the astronomical and geometric principles involved. The planning and coordination of the observations are no more involved than photographic parallactic observations of the moon would be, although there must of course be clear weather at all participating sites. The data, being in electronic form, are far more easily exchanged between participating sites than are photographs. Additionally, the necessary cooperation with students at other institutions adds the flavor of collaboration within a larger scientific community. Our experience has been that students are excited to be able to see a change in the sky (motion of the asteroid) in the course of a few minutes' time.

II. RANGE OF DISTANCES FOR WHICH PARALLAXES MAY BE DETERMINED

The fundamental concept in the parallax method of distance determination is that once any single linear dimension is known within a (physical or abstract) object whose geometric shape is unambiguously described, all linear dimensions can then be found. This of course is the principle used in surveying the surface of the earth, where precisely measured angles from two stations of known separation provide accurate distances to remote objects. In astronomy, the practical application of this concept has been used to determine the size of the earth in laboratory units (method of Eratosthenes),[3] the scale of the solar system in units of the size of the earth [e.g., transits of Venus[4] and other geometric methods of determining the earth–sun distance, known as the Astronomical Unit (AU)], and the distances to the nearby stars in terms of the AU. In practice, the parallactic shift of the apparent position of a distant object, as viewed from the ends of a baseline of known length, immediately yields the distance to the object through trigonometry. In most astronomical applications, the parallax angle is so small ($\ll 1$ rad) that a simple arithmetic formula is all that is needed to complete the calculation. In the case of parallactically determined stellar distances, to which most of our distance determinations in the universe are fundamentally tied and which are the subject of most textbook treatments of parallax, the angular shifts are very small—rarely more than, and usually much less than, 1″ (1 second of arc), even when observations are made from opposite ends of the earth's orbit. These are unfortunately far too small to be usable in student exercises. We concentrate, instead, on objects close enough to show a readily measurable parallax from observations made simultaneously from two sites separated by no more than a few hundred or thousand kilometers.

To help justify our choice of asteroids as likely parallax targets we introduce a few simple formulas which describe the overall scale of the problem in terms of the technology most likely to be applied to the problem. We assume at the start that an ideal laboratory exercise would involve the direct comparison of two overlapping images of the sky in which some object shows parallax with respect to the (distant) stars also visible in the images. The images might be displayed on a computer terminal, or might be hardcopy printouts. The relationship between the projected observa-

tional baseline B, the distance to the object Δ, and the parallax angle θ is given, in the small-angle approximation, by

$$\theta = \frac{B}{\Delta}$$

$$= 8.79'' \frac{B}{R_{earth}} \left(\frac{\Delta}{AU} \right)^{-1}$$

$$= 1.38'' \frac{B}{1000 \text{ km}} \left(\frac{\Delta}{AU} \right)^{-1}, \quad (1)$$

where R_{earth} is the radius of the earth (approximately 6370 km) and 1 AU is the mean earth–sun distance (approximately 150×10^6 km). Atmospheric blurring ("seeing") at the observing site will cause the angular sizes of the stellar images to be 2–5″, thereby limiting the effective angular resolution to approximately 1″, which is also a typical sky-projected CCD pixel size. For good quantitative results, therefore, an actual parallax of the order of 5″ or greater is desirable. From Eq. (1) we can see that with transcontinental baselines, solar system objects near the earth (within 2 AU) are potentially appropriate targets. Another useful relationship to keep in mind is that 1″ is one part in 206 265 of a radian, so that a projected baseline of 1/10 000 of the object's distance will show 20″ of parallax.

While Eq. (1) expresses a soft upper limit for measurable distances, there is also an effective lower limit, based on the finite field of view of any particular telescope/CCD combination. As most readily available CCD chips are relatively small, we can once again use small-angle approximations and express the linear field of view A of a telescope of aperture (diameter) D and focal ratio $f/$ (defined, in the usual photographic sense, as the ratio of the focal length to the aperture), used with a CCD chip of linear dimension W (equal to the product of the pixel width w and the number of pixels N), as

$$A = \frac{W}{D(f/)}$$

$$= 430'' \left(\frac{D}{0.40 \text{ m}} \right)^{-1} \left(\frac{f/}{12} \right)^{-1} \frac{W}{1.0 \text{ cm}}$$

$$= 430'' \left(\frac{D}{0.40 \text{ m}} \right)^{-1} \left(\frac{f/}{12} \right)^{-1} \frac{N}{500} \frac{w}{20 \, \mu m}. \quad (2)$$

The normalization values in this equation were chosen to be typical of instrumentation available for instructional purposes at many colleges and universities. Equation (2) indicates that expected parallaxes of up to several hundred seconds of arc may be used in principle, as there would then still be a good chance that a few reference stars would be common to the images from both sites. Otherwise, it would be necessary to resort to the computation of celestial coordinates in order to bridge across two nonoverlapping fields. As we will see in the next section, this problem exists for only a few rare, but interesting, cases.

Just as importantly, however, Eq. (2) indicates that in order for there to be a sufficient number of background stars for positional references, the images will have to be exposed long enough to record relatively faint stars (typically, 12th magnitude or fainter; that is, much fainter than visible to the naked eye). This is a difficulty, not because of lack of sensitivity—an exposure of well under a minute

Table I. Parallaxes for selected distances Δ and projected baselines B.

Asteroid distance, Δ	N/year	Projected baseline B					
		100 km	200 km Middlebury, VT to Hamilton, NY	500 km	1000 km	2000 km	5000 km east coast to west coast
0.01 AU	< 1	13.8″	27.6″	69.1″	138.″	276.″	691.″
0.05	≈ 1	2.76	5.53	13.8	27.6	55.3	138.
0.20	≈ 10	0.69	1.38	3.45	6.91	13.8	34.5
1.00	≈ 10²	0.14	0.28	0.69	1.38	2.76	6.91
2.00	≈ 10³	0.07	0.14	0.35	0.69	1.38	3.45

would normally suffice—but because of limited dynamic range (8 to 14 bits) in the CCD. The target itself will need to be of faintness comparable to that of the reference stars.

III. ASTEROIDS AS PARALLAX TARGETS

According to the criteria discussed in the previous section, asteroids (or *minor planets*) are excellent candidates for parallax measurements with CCDs on small telescopes on baselines of several hundred to several thousand km. The vast majority of known asteroids have orbits which confine them to the region of the solar system between the orbits of Mars and Jupiter (the "main belt"). At opposition, which occurs when the asteroid and the sun are in opposite sides of the celestial sphere, such an asteroid will typically be at an earth distance Δ of between 1.2 and 2.5 AU. There are several hundred of these whose mean opposition visual (V) magnitudes are V = 13 or brighter; roughly a thousand occasionally appear brighter than V = 16. Computer database-assisted searches of the asteroids whose orbits are sufficiently well determined that their positions may be confidently predicted suggest that, at any time, we expect at least a dozen asteroids which will show parallax of about 3″ or more on a transcontinental baseline, with visual magnitude 13 or brighter, and within 15° of the antisun (that is, near opposition and therefore reasonably placed in the sky for simultaneous observation from widely separated sites). As a more concrete example, in the 1990 edition of *Ephemerides of Minor Planets* (EMP)[5] there are 17 asteroids which reached opposition in Jan. 1990 at visual magnitude 13.0 or brighter and at an earth–asteroid separation of 2.0 AU or smaller. Furthermore, there are many asteroids which approach the earth much more closely and therefore show parallaxes observable on much shorter baselines. With the Spacewatch Telescope[6] CCD system now operational in addition to other ongoing asteroid surveys,[7] we can expect several discoveries per year of asteroids which will pass within 0.5 AU and reach magnitude 16 or brighter.

Table I gives some specific parallax figures for several combinations of Δ and B. The range in distance Δ tabulated covers 0.01 to 2.0 AU, a range which includes at least occasional opposition passages of the thousands of known main-belt asteroids. (The record for close approach as of this writing, by 1991 BA in Jan. 1991, is just over 0.001 AU,[7,8] about half the distance to the moon.) The range of projected baselines includes, at the low end, 100 km, about half that for which successful parallax observations have been made already (between Colgate University and Middlebury College), and, at the high end, the baseline available in North America in an east coast–west coast collaboration. The second column gives, for each of these distances, the order of magnitude of the number of opportunities per year that any observatory equipped with a CCD on a small telescope can expect to have. Note that an asteroid would typically be at these distances for several weeks. Therefore, we can conclude that on nearly any night of the year there will be several asteroids, the observation of which would show parallax of at least a few arcseconds for collaborations including both east coast and west coast observers, and that several times per year there will be an opportunity for similar results from observations of near-earth approachers (NEAs) from a more limited geographical region.

The fact that these observations will always depend on our knowledge of the asteroid's orbit should not be viewed as detrimental to the value of the exercise. An asteroid's orbit is usually expressed mostly in angular parameters, the linear scale being given in AU. All of this information is derivable from earth-based observations and does not depend on our knowledge of the size of the AU in laboratory units, such as meters. (The necessary algorithms to perform these calculations, and their implementation in computer programs, are widely available[9–11] but this exercise will not be considered in this paper.) Therefore, the observation of the parallax of an asteroid over a baseline whose length *is* known in laboratory units amounts essentially to a determination of the size of the AU, and should be an integral part any laboratory exercise based on such observations. In fact, even as late as the 1950s,[12] this was one of the methods by which our most precise knowledge of the AU in laboratory units was learned; other techniques, such as radar ranging of Venus, have since provided even higher precision.

IV. DIRECT PARALLAXES OF ASTEROIDS

In this section we will discuss logistical matters and consider the means of analysis of the data. We assume that the potential participants understand how to operate their telescopes and CCD cameras, are reasonably adept at locating faint objects in the sky, probably through the use of some combination of telescope position readout and finding charts, and are familiar with Universal Time (UT) and the standard celestial coordinates, right ascension (α) and declination (δ). The conceptually cleanest version of this exercise is one in which two or more images of the same asteroid are obtained *simultaneously* from two or more widely separated geographical locations.

Table II. Orbital and observational parameters for a sample of NEA and mainbelt asteroids.

Asteroid	a	e	1992 March 20				
			Δ	V	$\dot{\alpha}^{a}$	$\dot{\alpha}\cos\delta$	$\dot{\delta}$
1991 TB$_1$[b]	1.455AU	0.353	0.15 AU	14.2	−0.078"/s	−0.068"/s	−0.120"/s
1992 AC[b]	2.104	0.422	0.38	13.9	0.018	0.010	−0.005
(1423) Jose	2.669	0.114	2.08	14.7	−0.008	−0.008	0.003

[a]West-to-east rate projected to celestial equator.
[b]Based on preliminary orbital elements (as of 10 March 1992).

Selection of the potential target asteroid may be made in several ways, all of which have proven useful to the authors. (1) A search may be made, for a given date, through a database[13] of asteroid orbital elements and magnitude parameters. This would be comprehensive to the extent that the database is complete, but requires a method for performing calculations of ephemerides[14] (sequences of predicted celestial coordinates, based on the orbital elements). This is perhaps a task that could only need to be performed centrally, with the results to be disseminated among all those interested. (2) Close passages of note are often published in specialized journals, e.g., The Minor Planet Bulletin, and occasionally in articles in the main scientific journals.[15] (3) As mentioned above, simple perusal of the *Ephemerides of Minor Planets*, for which the requisite information is organized chronologically, will turn up many excellent opportunities from among the numbered asteroids. The best prospects are likely to be near opposition, which is also when they will be at their highest altitudes during the middles of the nights of both sites. This will allow observation at times for which the projected baseline (component of the separation of the two sites perpendicular to the line of sight to the asteroid) will be as large as possible. (4) Discoveries of, and ephemerides for, newly discovered NEAs are announced in the International Astronomical Union (IAU) Circulars, subscriptions to which are available through U.S. mail and electronic mail.[16]

It is necessary to have accurate ephemerides for the target asteroid. In extreme cases, separate calculations for different geographical locations may be called for, but this would be unusual. It is not out of the question, however, for the known orbital elements to be sufficiently in error that the actual and predicted positions at a given time will be discrepant by a large fraction of the CCD field of view or more, especially for newly discovered NEAs. Therefore, it would be prudent for some of the participants to attempt to locate such objects a day or two ahead of time, so that any discrepancies can be taken into account on the night of the actual experiment. The brightest asteroids generally have well-enough determined orbits so that this precaution is unnecessary. The motions of the asteroids among the background stars can be very rapid, especially for those passing close to the earth. The rate of motion may be great enough for a main-belt asteroid at opposition to carry it across the field of the CCD camera in less than a day, with correspondingly greater speeds for NEAs; therefore, it is convenient to have ephemerides calculated at a high resolution in time, perhaps even at hourly intervals.

The observations themselves would in most cases have to be closely coordinated to ensure simultaneous midexposure times. The exposures need not however be of equal duration. One method is to maintain telephone contact throughout the observations; if three or more sites are participating, however, it may be necessary to agree in advance on one or more target epochs. The epochs can be chosen during the observing session, perhaps by a designated "coordinator site," and broadcast to the other participants by electronic mail, or even by a series of brief phone calls. The degree to which the exposures need be simultaneous depends on how rapidly the asteroid is moving and on what the expected parallax should be for the given baseline; both of these data are readily known from the asteroid's ephemeris. For example, if an asteroid at a distance of 0.5 AU is moving at the rate of 0.1" per second of time, and the available baseline should produce a parallax of 10", two observations offset by 10 s would produce a considerable, though not disastrous, error; a timing error of only 1 s would produce an error which is likely to be less important than errors introduced during astrometric analysis of the images. Table II illustrates the rates of motion, in "/s, on 20 March 1992, of the NEAs 1991 TB$_1$ (at a distance of $\Delta \approx 0.15$ AU) and 1992 AC ($\Delta \approx 0.38$ AU), and of the main-belt asteroid (1423) Jose (near opposition, at $\Delta \approx 2.08$ AU), according to their orbital elements as known at that time. The second and third column list, respectively, the semimajor axis and eccentricity of the orbits, while the third and fourth columns list the predicted distances from the earth and visual magnitudes, for 1992 March 20 0^h UT. The sixth column, headed $\dot{\alpha}$, gives the rate in the west-to-east direction as projected to the celestial equator, while the last two columns give the predicted west-to-east and south-to-north rates of motion on the sky. The figures in the last two columns may be considered to be Cartesian coordinates of the proper motions of the asteroids, from which their angular speeds and directions may be obtained directly.

Assuming that two or more images are obtained essentially simultaneously, so that the entire positional shift can be attributed to true parallax, rather than asteroidal (orbital) motion across the sky, it is then relatively straightforward to convert this information to a parallax angle. The exercise proceeds best by having hard copies of both images at a common printed scale. These are overlayed, matching up the reference stars, to show directly the parallactic shift, which then may be measured with a ruler. Auxiliary images with the same telescope system(s) of two stars of known angular separation can be used to determine the image scale (e.g., "/mm), which then is used to convert from linear distances in the images to angular separations. Note that in this method, it is not necessary for the CCD pixel rows and columns to be accurately aligned with the N–S and E–W directions on the sky. With moderately sophisticated image display software and an associated
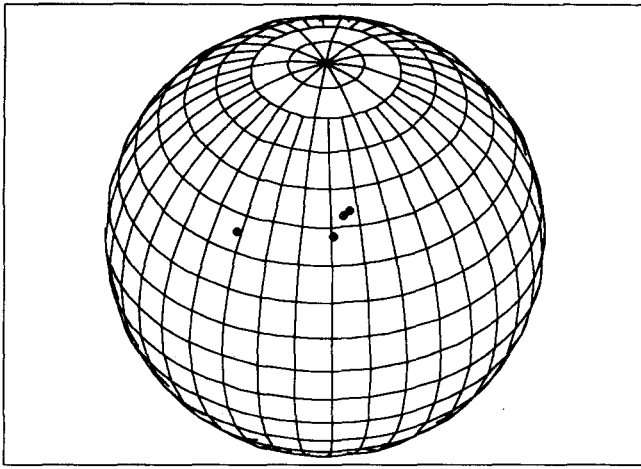
Fig. 1. A map of the surface of the earth which could be used for determining the projected baseline as a fraction of the earth's radius, created by a commercial microcomputer software package.[17] This particular map shows (circled dots) the geographic locations of, left to right (west to east), Flagstaff, AZ, Lexington, VA, Hamilton, NY, and Middlebury, VT, the four sites from which the parallax observations discussed in Sec. V were made. The map shows the orientation of the earth as it would have appeared from asteroid 1991 TB$_1$ on March 17, 1992 at 6:15 UT, the time at which the images shown in Fig. 2 were made.

printer, or with any printer and a reducing/enlarging photocopier with nearly continuous reproduction ratio, printed copies of the images can be produced at the same linear scale.

An alternate method exists which eliminates the need for direct printing of the images: The Cartesian display coordinates of the reference stars and asteroid can be measured for each image by reading the pixel values of the object center from the computer display using the cursor (a technique available in all CCD systems or image display programs with which we are familiar). The relative scale factor between the images obtained at the two sites can be determined by using a pair of reference stars visible in both frames, using the Pythagorean theorem. The data $(x,y$ coordinates for each star or asteroid) can then be scaled by this factor and plotted by hand or with a simple computer graphics program, thereby producing maps, to a common scale, of the objects in each image. These maps may then be overlayed as in method described previously. This second technique has the advantage that students at the two sites can exchange this information by phone or electronic mail almost immediately after the images are obtained rather than having to wait for, and to process, images from the other site.

The most difficult part of this version of the exercise would then be the calculation of the projected baseline for use in Eq. (1). The method used would depend on the level of the students participating in the exercise. For observations made within 30° of the zeniths of both observing sites, the foreshortening of the baseline amounts to no more than 15% and could well be ignored or approximated. More generally, the baseline can be calculated with additional visual aids, taking into account the observatories' geographical locations and the observed celestial position of the asteroid. Figure 1 shows a computer-generated[17] view of the earth as seen by an asteroid, with the locations of several observatories marked as they would appear at the

time of observation. Measurements on the figure and the knowledge of the earth's diameter then leads directly to the projected baseline. Another, equivalent method involves marking the observatory locations on an actual globe, orienting the globe as seen from the direction of the asteroid, and measuring the projected separation between sites as a fraction of the globe's (earth's) diameter. While of potentially low accuracy, this perfectly general method has strong pedagogical appeal.

The more careful analysis, suitable for science majors, would reinforce concepts from geography, spherical coordinates, vector arithmetic, and spherical trigonometry. A realistic mathematical model of the earth would be used to calculate accurately the baseline vector and the direction vectors from the observing sites to the asteroid, from which the projected baseline follows. Image processing routines would be used to find precise pixel positions of the stellar and asteroidal images and astrometric reduction programs would be utilized to perform the conversion from rectangular to celestial coordinates. Such a rigorous analysis would substantially reduce inaccuracies of measurements compared to the overlay method, while simultaneously providing the basis for formal uncertainty analysis. Details of the application of this standard astrometric exercise, especially to asteroid observations, can be found elsewhere.[18-20]

V. EXAMPLES: OBSERVATIONS OF THE PARALLAXES OF 1991 JX, (3103) 1982 BB, 1991 TB$_1$, AND 1992 AC

The authors of this paper, in various combinations, have on several occasions organized and performed parallactic observations of asteroids, and much of what we have outlined above was conceived in the course of planning these attempts. Our experiences in the execution of these observations, to be described in this section, illustrate some of the difficulties and rewards of parallax measurements. During the actual observations, some additional techniques proved valuable for ensuring that the resultant images would be as useful as possible. These include: using open telephone connections to ensure that the midexposure times were simultaneous to within approximately 1 s; preferentially taking exposures when the asteroid was relatively near several reference stars; and choosing exposure durations just long enough to adequately detect the faint reference stars.

Successful coordinated parallax observations of two asteroids (to our knowledge, the first ever made) were completed in June 1991 for the asteroid 1991 JX, and in July 1991 for the asteroid (3103) 1982 BB.[18] Each of these asteroids was, at the time of observation, well under 1.0 AU distant. The baseline was 232 km, between Colgate University in Hamilton, NY and Middlebury College in Middlebury, VT. Because of equipment problems during the observations of 1991 JX, and because of the relatively short baseline available on those both occasions, the images obtained were not suitable for quantitative analysis by the overlay method. Nonetheless, the observations clearly indicated that even on such a short baseline, a prudent choice of target can yield excellent results.

1991 JX was discovered 9 May 1991, and it was soon clear that it would pass very close to the earth, remaining within 0.1 AU for several weeks.[21] Accordingly, it was

observed with 0.4 m reflecting telescopes at Colgate University's Foggy Bottom Observatory (FBO) and at the Middlebury College Observatory (MCO) on 8 June (UT), 1991. Both telescopes were equipped with CCD imaging cameras of approximately 500 pixels on the side. The predicted distance was only 0.038 AU, which according to Eq.(1) would yield a parallax of as great as 8″. Unfortunately, electronics problems with the MCO CCD system caused those images to be badly smeared, rendering quantitative analysis difficult and the results imprecise. In spite of the equipment problems, the parallax was evident by the overlay method and the numerical results were consistent with the predicted distance.

Asteroid (3103) 1982 BB was discovered 20 January 1982.[22] Its orbit has been determined so well that it has been "numbered," hence the designation (3103). It is very nearly in a 5:3 orbital resonance with the earth, and passes close to the earth with regularity.[23] Its close passage in the summer of 1991 provided another opportunity to further develop our techniques. On 17 July (UT), 1991, 11 pairs of simultaneous images were obtained at FBO and MCO over a period of about 2 h. The predicted distance of 0.224 AU, or 33.6 million km (varying by less than 1% during the interval of the observations), indicated a parallax of less than 2″, just barely detectable by the overlay method. Careful image processing and formal astrometric reduction of the stellar and asteroidal positions[18,19] yielded a distance of 32.2 million km ± 5%, which agrees with the predicted distance (based on the known size of the AU in km) to within the calculated uncertainty.

The first images suitable for parallax measurement using the overlay method were obtained in mid-March of 1992, as the result of a collaboration between Colgate University, the Virginia Military Institute (VMI; Lexington, VA), the National Undergraduate Research Observatory (NURO; Flagstaff, AZ), and the U. S. Air Force Academy (Colorado Springs, CO). Our primary target was asteroid 1991 TB₁ (discovered 10 October 1991[24]), which was predicted to be about 0.14 AU distant from the earth at the time of the planned observations.

Because the orbital elements for 1991 TB₁ available at the time of this effort were based on only a 1-month interval of observations from late 1991, we anticipated difficulties in locating it. Observers at Colgate and VMI spent parts of two nights searching near the asteroid's predicted position, sometimes using the techniques of taking exposures of several minutes duration while tracking the telescope at the expected rate of motion of the asteroid, and then looking for the round image of the asteroid among trailed stellar images. On 14 March (UT), the asteroid was found at VMI by this technique,[25] about 1/4° (wider than our CCDs' fields of view) from its predicted location. That same night, two pairs of simultaneous images were obtained of the asteroid using the 0.5 m VMI and 0.4 m FBO telescopes; deteriorating weather prevented further work. The next night (15 March), using revised celestial coordinate predictions based on the previous night's observations, the asteroid was easily found at VMI and with the NURO 0.8 m telescope, and two pairs of simultaneous images were procured at these two sites, over the baseline of roughly 3000 km between Flagstaff and Lexington. Two nights later, on 17 March (UT), four pairs of simultaneous images of the asteroid were obtained at VMI and NURO.

1991 TB₁ moved rapidly, crossing the field of view of the
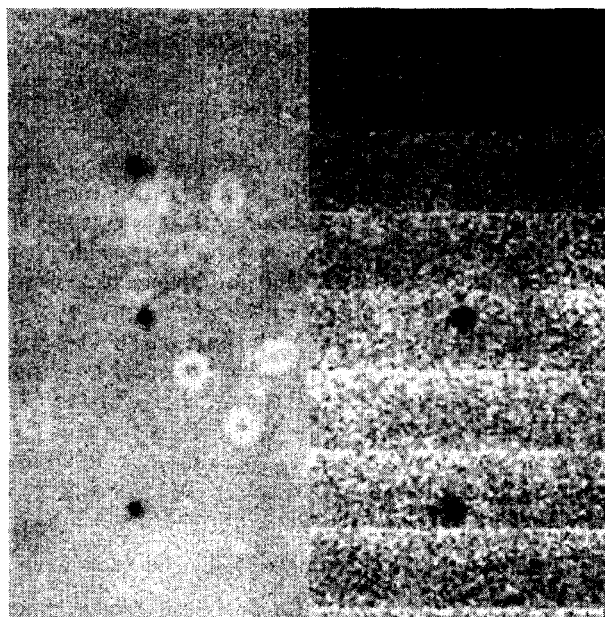

Fig. 2. Two images of asteroid 1991 TB₁, made simultaneously on March 17, 1992 at 6:15 UT from the observatories of NURO in Flagstaff, AZ (left panel) and VMI in Lexington, VA (right panel). In each half of the frame, reproduced as a negative image, the central and lower round spots are two distant stars; the upper spot is the asteroid and shows a very large parallactic displacement. The asteroidal images are trailed diagonally, due to the asteroid's motion during the exposures (see Table II). In this figure, north is at the top and east is to the left; the parallax is almost entirely in the east–west direction, as would be expected from the relative geographic locations of the two observatory sites (see also Fig. 1.) Low-level structure (rings, graininess, and horizontal streaks) can be seen in both panels, and is due to a combination of dust on the optical elements, intrinsic pixel-to-pixel variations in sensitivity of the CCD chips, and artifacts of the camera readout electronics. For photometric observations, these variations would be removed with calibration frames, but that image processing step is clearly unnecessary for this parallax exercise.

NURO CCD chip in less than an hour, and trailing of the asteroid was visible even in relatively short exposures. To adequately record the reference stars, however, exposures of duration 30 s were made at NURO, and of 40 s at VMI. Figure 2 shows parts of two simultaneous images of 1991 TB₁ from the VA and AZ sites, made at 6:15 (UT) on 17 March 1992. The parallax of the asteroid (upper of three obvious spots in each panel) is approximately 25″ and is predominantly in the east–west direction, as expected. The images of the asteroid are slightly elongated, due to the asteroid's motion during the exposures.

On 17 March, VMI and NURO also obtained six pairs of simultaneous images of asteroid 1992 AC (discovered 5 January 1992), which was expected to be somewhat less than 0.4 AU distant.[26] 1992 AC was found very close to its predicted position, and due to its slower rate of motion (see Table II) it was easier to observe. Although the observed parallax was smaller than 1991 TB₁, the slower motion allowed for longer exposures which in turn yielded more reference stars in each frame.

In spite of the potential availability of four telescopes, imperfect weather conditions and the observers' travel schedules prevented us from making observations from more than two sites at any one time during the week-long campaign. Therefore, we were unable to attempt the more

spectacular, but logistically more difficult, exercise of producing images from three or more sites simultaneously.

VI. OTHER, RELATED EXPERIMENTS

While the experiment we have outlined above is the cleanest conceptually, it involves some potentially stifling constraints. The first is logistical: The cooperation of a distant observing site may not always be possible to arrange. The second is the fact that the lower limit of measurable parallax (determined to be a few seconds of arc both by typical pixel sizes and, to some degree, by atmospheric seeing) differs by only a few orders of magnitude from the upper limit (set by the limited size of the CCD chip, typically 500 pixels on a side). Thus for any given baseline, the range of distances which may be determined also spans just a few orders of magnitude. Therefore, we would like to mention a few variations on the theme.

A. Single-telescope observations of asteroid parallaxes

If the orbit of the asteroid were known exactly, then over the course of a night of observations, slight discrepancies in the position of the asteroid would be observed, due to topocentricity of the observations. That is, the rotation of the earth would provide a large baseline, except near the north and south poles. As we have pointed out, however, an asteroid's orbit is rarely known exactly, so that it would not be possible to know with absolute certainty how much of the asteroid's apparent motion is due to parallax, and how much is due to orbital motion. Here are two strategies for dealing with this difficulty.

1. Asteroids near stationary points

Main-belt asteroids usually show normal retrograde loops; at either end of the loop (a few months before and after opposition), the asteroid briefly shows essentially no motion in right ascension. Therefore, any perceived motion in right ascension can be attributed purely to parallax. Drawbacks to this method are several in number. There are very few nights per year for which any given asteroid has a sufficiently small motion in right ascension. An asteroid near a stationary point, unless it is at a favorably extreme declination, will only be observable for part of the night; and a main-belt asteroid near its stationary point will in general be more distant than at opposition. Nonetheless, several observations of such an event, spread out over 5 or 6 h in late evening or early morning, would yield a parallax of 2–3″ for a main-belt asteroid, and even more for an NEA which shows a well-defined stationary point.

2. Asteroids near opposition

The difficulties inherent in the single-telescope stationary-point experiment just described are eliminated, at the cost of requiring several observations to be made on two consecutive (or nearly so) nights, by observing an asteroid as close as possible to its opposition passage. For main-belt asteroids, and for many NEAs as well, the asteroid at will be closest, brightest, and observable for the largest fraction of the night, when it is at opposition. At first glance, there is the difficulty that this is the time of greatest motion in right ascension, which would utterly swamp the parallactic motion expected from the baseline induced by the earth's rotation. However, at opposition, the rate of the orbital motion in right ascension is typically

sufficiently uniform that observations on two consecutive nights will allow this mean motion to be subtracted out, leaving only the superimposed parallactic motion. (This is to be preferred to using the known orbit to calculate the orbital rate, but this alternative method might suffice in the case of a very accurately known orbit, which would thereby make a single-night, single-site exercise feasible.)

B. Single-site multitelescope parallaxes of artificial earth satellites

The availability of two or more small, CCD-equipped telescopes at one site would make possible the measurement of parallaxes of artificial earth satellites. As will be explained, the images would be difficult to analyze by conventional astrometric techniques, being a combination of trailed and nontrailed images. Therefore, the preferred method of analysis would be the "overlay" method, so that the fields seen by the two telescopes must be overlapping.

1. Parallaxes of low earth-orbit artificial satellites

Satellites in low earth-orbit (altitudes 200–500 km) require projected baselines of much less than 1 km. For example, for a satellite at a distance of 500 km = 500 000 m, a 50 m baseline will provide a parallax of approximately 20″! In fact, for such a satellite passing nearly directly overhead, the best observing strategy does not even require exact simultaneity of the exposures: Low orbit satellites move so rapidly when near the zenith (several degrees per minute) that most astronomical telescopes cannot even follow them. However, at favorable passages these satellites are also so bright when illuminated by sunlight that the only feasible way to record them AND the background stars needed for reference, is to allow the telescope to track the sky normally, expose for however long is necessary to reach the field stars, and allow the satellite to be recorded as a long trail through the image. If either the beginnings or the ends of the two exposures are exactly simultaneous, then the ends of the recorded trails could be used to represent the satellite's position. Given the difficulty of predicting the exact paths of these satellites, and the requirement that the exposures be started or stopped simultaneously to a small fraction of a second, it is not recommended that such simultaneity be attempted. Instead, the telescopes should be arranged, or the satellite should be chosen, so that a significant component of the displacement between the two telescopes is perpendicular to the direction of the satellite's path. This component is then compared to the displacement between the two recorded trails, as indicated by the background stars, to compute the distance to the satellite. General information about satellite observations, including predictions based on orbital parameters, can be found elsewhere in the literature.[27,28]

An alternate technique would be to use CCD cameras with short focal-length telescopes or ordinary photographic lenses, providing a much larger field of view at the minor cost of requiring a larger baseline. For any particular focal length lens, Eqs. (1) and (2) can be used to determine what baseline would be needed in order to provide a parallax of several pixels. This method has the advantage that the need for high-accuracy satellite predictions is greatly reduced.

2. Parallaxes of geosynchronous satellites

Satellites in geosynchronous orbits can also have their parallaxes detected by a pair of CCD-equipped telescopes at an appropriate separation. As their distances are approximately 40 000 km, a parallactic displacement of 10″ requires a projected baseline of about 2 km, which, for a geosynchronous satellite observed toward the south from the continental U. S., translates to a north–south displacement of approximately 3 km. Geosynchronous-orbit satellites, which are faint (typically 12–14th magnitude[29]), are essentially fixed with respect to the earth's surface, so that they move at approximately 15″/s against the background stars. Therefore, the best observing strategy would be to turn off the telescope drive and let the stars trail through the image. As for the low-earth satellites, exact simultaneity of exposure is unnecessary, and the north–south component of the baseline will be reflected in the displacement, between the images from the two sites, of the pointlike image of the satellite relative to the trailed stellar images.

VII. CONCLUSIONS

The CCD imaging camera promises to revolutionize the educational and scientific capabilities of small telescopes. The digital data obtained on a rectangular grid are well suited to the development of astrometric exercises; and even for introductory level students, a valuable parallax exercise can be devised in which the students can obtain and analyze the data without having to resort to tedious image processing or to mathematical analysis beyond trigonometry. Such exercises reinforce the concepts of stellar parallax, help build a foundation for future work in photometric image processing and/or astrometric reduction to celestial coordinates, and can foster cooperation between educational institutions. Between asteroids and earth-orbiting artificial satellites, there are plentiful targets appropriate to nearly any available baseline, particularly among asteroids observed over baselines of 1000 km or more. The ease with which this idea spread from its original conception (by CRP and LAM) to the several successful observations mentioned above leaves us with no doubt that this experiment will become routine.

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Negative mass can be positively amusing

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Some insights into gravitation and mechanics, as well as some surprises, result from considering the dynamics of negative mass particles.

Negative masses may be unphysical, but several examples are given of how they can be instructive. In these examples, the dynamics and gravitational interactions involving negative mass particles produce surprising (and amusing) predictions about the motions of the particles. That these apparently impossible motions are, in fact, compatible with physical laws leads to some insights about dynamics and gravitation.

At the outset it must be made clear that the "mass" of a body has several different roles in mechanics: (i) The "inertial mass" of a body refers to its resistance to acceleration; it is the m in $F=ma$. (ii) The "gravitational," or "passive gravitational mass," governs how strongly gravity pulls on the body; it is the m in $\mathbf{F}=-m\nabla\Phi$, where Φ is the gravitational potential. (iii) There is also "active gravitational mass," which determines the strength of the gravitational field generated by a body.[1] In standard physical theory all three "masses" are identical.

The equivalence of inertial and passive gravitational mass, in particular, has been verified to high accuracy, and is known by the name "the principle of equivalence." It is this principle that allows us to cancel the masses in Newton's second law, and in the force law for gravity, so that the gravitational acceleration is given by $\mathbf{a}=-\nabla\Phi$. Since no reference to the mass appears, this equation predicts that (in a vacuum) feathers and rocks fall in the same way. We will assume that the equivalence principle holds, so that our negative mass particles have both negative inertial mass, and negative passive gravitational mass.

The first situation to be considered is the fall of a negative mass particle when it is released from rest. The particle, like rocks and feathers, must of course fall downward.[2] For the negative mass particle, of course, the gravitational force on the particle is upward. The particle accelerates (downward) in the direction opposite to the force acting on it due to the particle's negative inertial mass. More interesting than the free-fall of the negative mass particle is how it must be constrained, e.g., before it is released, to prevent it from falling. Since the gravitational force on it is upward, the support force to prevent the particle from falling must be downward. For example, the particle could be tethered by a string and the other end of the string could be pulled downward. This would give us a child-with-a-balloon configuration, with the negative mass particle suspended above the child, who feels an upward pull on the string. There is, of course, a crucial difference between the negative mass particle and an actual balloon. If the string breaks, a balloon—to the child's chagrin—would accelerate upward, whereas the negative mass particle would fall downward.

This becomes even stranger if we replace the child as the agent of downward force by a positive mass particle. Suppose that we have a negative mass particle of -1 kg at the top of the string in the earth's uniform field. Let us put a $+1$-kg mass at the bottom of the string. The $+1$-kg mass pulls downward on the string with a force of 9.81 N, the -1-kg pulls upward on the string with the same force. The string remains under a tension of 9.81 N and the whole configuration—positive mass, negative mass, and string—remains fixed in position falling neither downward or upward. We have created an "antigravity glider." This is acceptable, if not sensible, since the total mass of the configuration is zero.[3] But the consequence of cutting the string may be less acceptable. If the string is cut both particles fall!

We can further offend our intuition with the antigravity glider. Suppose we release from rest the ±1-kg mass configuration with 10-N tension in the string. At the bottom of the string the 10-N upward tension force will not quite cancel the downward 9.81-N gravitational force. There will be a net upward force of 0.19 N which will result in an upward acceleration of the $+1$-kg mass by 0.19 m s^{-2}. At the top of the string, the 10-N downward tension and the upward 9.81-N gravitational force leave a net 0.19-N