RADAR MEASUREMENT OF THE ROTATION RATE OF MERCURY

Student Manual

A Manual to Accompany Software for
Introductory Astronomy Lab Exercise
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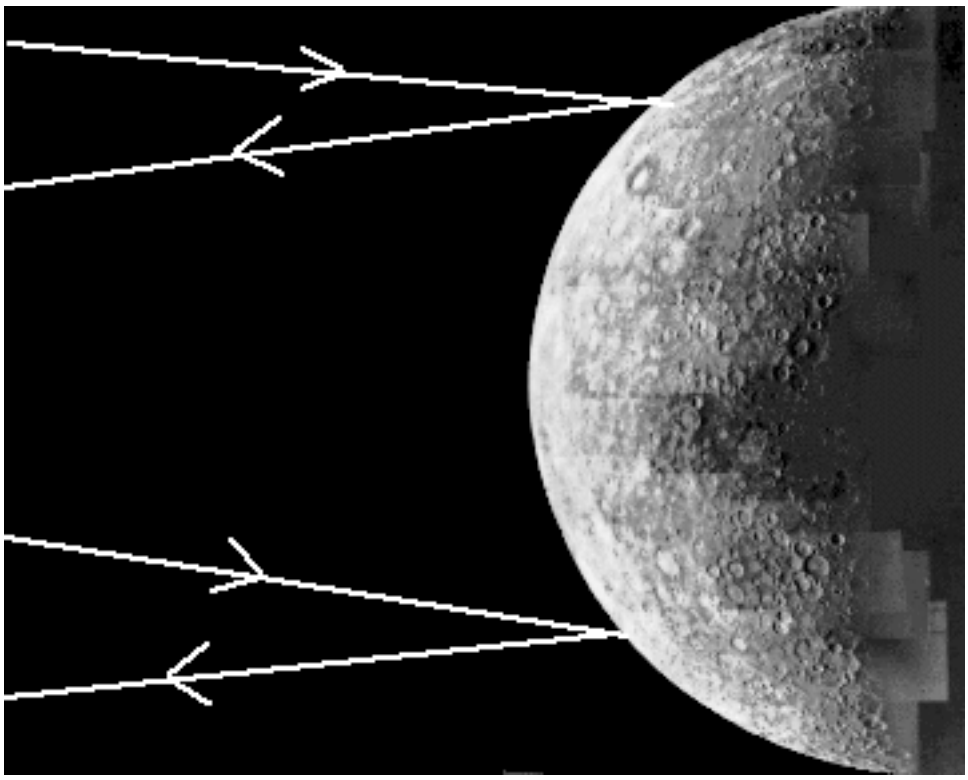

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Contemporary Laboratory
Experiences in Astronomy

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Goals:

You should be able to measure the broadening of the reflected signal from a rotating body to determine the rate at which it rotates.

Objectives:

If you learn to .....

Use a simulated radio telescope to acquire pulse spectra.

Read a pulse spectrum to find the frequency shift of the pulse.

Measure the Doppler-shift to interpret the change in frequency between the outgoing pulse and the reflection.

You should be able to .....

Determine the radial velocity of Mercury and calculate its period.

Calculate the rotational period of Mercury.

EQUIPMENT

Sky and Telescope Reprint LE 11, pocket calculator, IBM-compatible computer running CLEA software for "Radar Measurement of the Rotation Rate of Mercury".

INTRODUCTION

Because Mercury is a small planet whose surface features have low contrast, and because it is so close to the sun that it is rarely visible against a dark sky, it is difficult to determine how fast it is rotating merely by looking at it from the earth. In recent years, however, radar techniques have proven most effective in measuring its speed of rotation. The method you will employ here actually has wider application than just the measuring the rotation of Mercury. It can be used to study the behavior of other planets as well, from cloud-covered Venus to the rings of the major planets, to the smallest asteroids.

The technique of this lab is described below and in Reprint LE 11 by Hoff and Schmidt. The basic idea is to use a radio telescope to send short pulse of electromagnetic radiation of known frequency towards the planet Mercury and then to record the spectrum (frequency versus intensity) of the returning echo. Depending on the relative position of the earth and Mercury, the pulse will take between 10 minutes and half hour to travel to Mercury, bounce off, and return.

By the time the pulse has reached Mercury, it has spread out to cover the entire planet. Because the planet's surface is a sphere, the pulse hits different parts of the planet at different times, however. The pulse first hits the surface at a point directly on a line between the centers of the earth and Mercury (the "sub-radar point"), and few microseconds later from points further back, toward the edges of the planet. Thus we wait for the first echo, from the sub-radar point, and then by looking at the returning echoes at succeeding times, each a few microseconds later than the next, we get information about different parts of Mercury's surface.

The frequencies of the returning echoes are different from the frequency of the pulse sent out because the echoes have bounced off the moving surface of Mercury. Any time a source of radiation is moving radially (towards or away from the observer) there will be a Doppler shift in the received frequency that is proportional to the velocity along the line of sight.

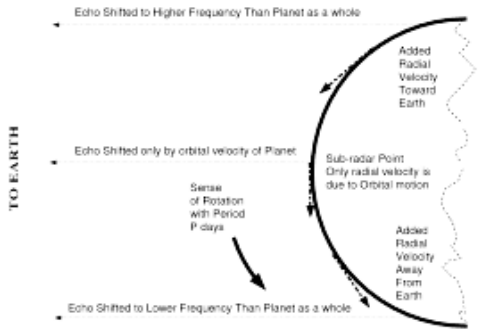


FIGURE 1

There are two motions of Mercury that can produce such a shift: its orbital velocity as a whole around the sun, and its rotation around its axis. The first echo, from the sub-radar point, is shifted in frequency only by the orbital velocity of the planet as a whole. We can calculate how fast the planet is moving with respect to the earth from the amount of the shift, but we can't tell how fast it's rotating (spinning) because the component of the rotational velocity of the surface of Mercury is perpendicular to our line of sight at this point (see Figure 1), and so there is no additional frequency shift. The echoes that come in after the sub-radar echo, however, show additional shifts because they come from further back where the rotational velocity is more directly along our line of sight. Because of the rotation of Mercury, one edge of the planet is moving a towards us a little faster than the planet as a whole, and the other edge is moving away towards us a little slower than the planet as a whole (See Figure 1). So, due to the Doppler effect, part of the returning echo (from the faster moving edge of Mercury) is at a slightly higher frequency, and part of the returning echo (from the slower moving edge) is at a slightly lower frequency as shown in Figure 2:
We measure the amount of this frequency shift and apply our knowledge of the Doppler effect to calculate the velocity of the surface of Mercury, and from this, its period of rotation.

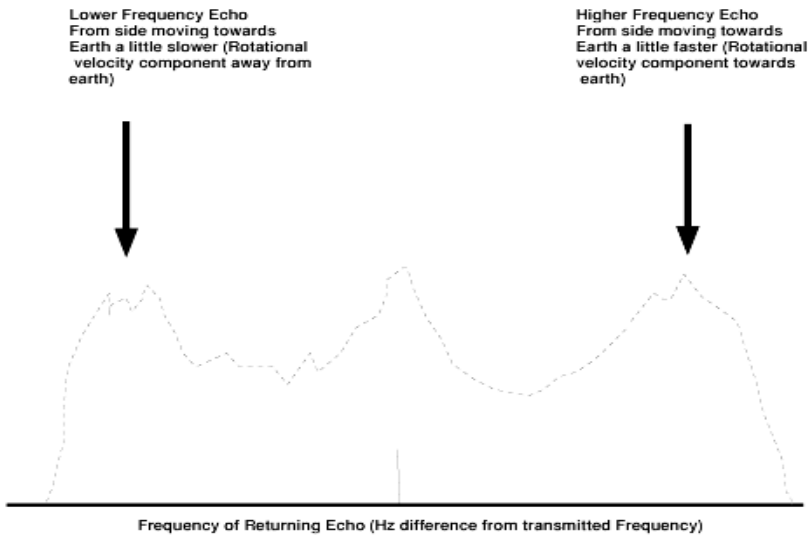


Figure 2

Before you proceed any further, please read *Sky and Telescope* reprint LE 11 carefully to get an idea of the details of the method. Then answer these questions:

1) What was the earliest method used to determine the rotation period of Mercury?

2) What was the difficulty of applying this method?

3) Until the radar measurements, what was the accepted rotation period of Mercury?

USING THE CLEA RADIO TELESCOPE

Overall Strategy

This is the overall plan of action for this lab:

1) Turn on radio telescope.
2) Calculate position of Mercury and point telescope to it.
3) Send out pulse.
4) While waiting for echo to return, calculate geometrical parameters for echoes at 120, 210, 300, and 390 microseconds after the arrival of the sub-radar pulse.
5) Measure the highest and lowest frequency shift of the returning echoes from the four delayed echoes.
6) Record these data and then use them to calculate the rotation velocity of the equator of Mercury from each of the four delayed echoes.
7) Calculate the rotational period in days from the velocity and the known circumference of the planet.
8) Check answers for reasonableness and make sure that all requested information is provided.

Getting Started

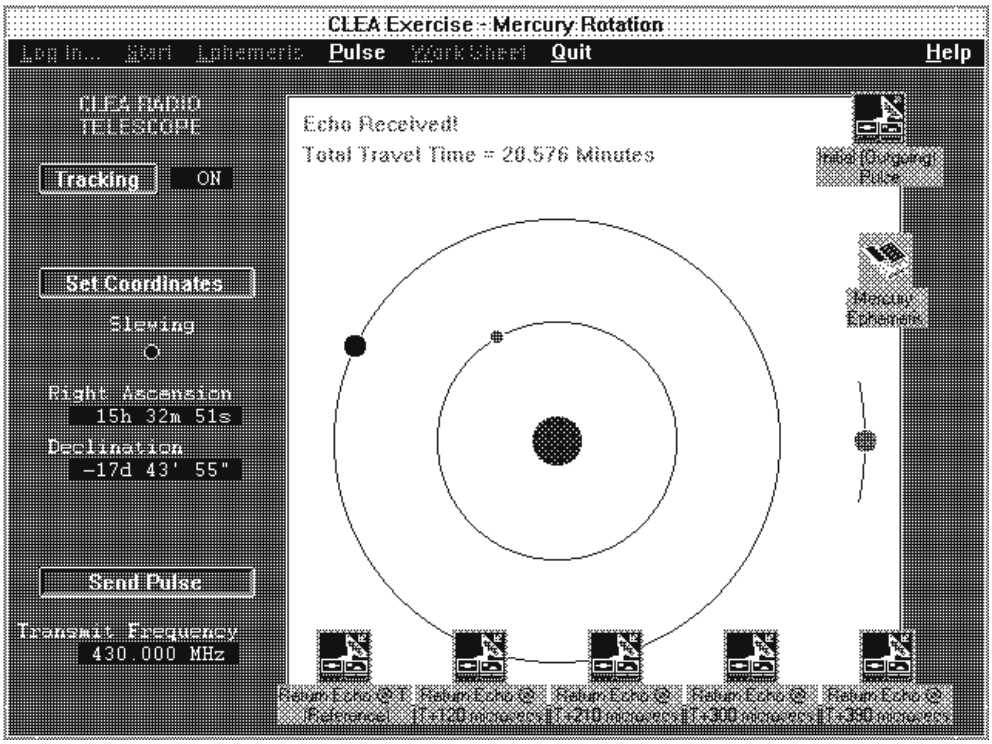
This exercise simulates the operation of a large radio telescope used to determine the rate of rotation of a planet by Doppler radar. Begin by selecting "Log In" on the main menu, and fill in the requested information. After completing the log-in, select "Start" on the main menu. The control panel that appears has only three push-button controls, as well as displays to show the frequency the telescope is tuned to and the coordinates it is pointed to in the sky. Begin by pressing the "Tracking" button to turn on the sidereal drive so that the telescope will track the planets as the earth turns. This will activate the other controls menu.

If you already know the coordinates and the distance from the Earth to the target planet, press "Set Coordinates", respond "OK" to the message that appears, and enter the coordinates and distance on the dialog box that appears. When you have finished press "OK" and the telescope will begin moving to point to the target. In most instances, however, you will not have a set of planetary coordinates. If this is the case, do not press "Set Coordinates" yet (or if you have already done so, press "Cancel" on either the message or dialog that follow). Select "Ephemeris" from the main menu; this activates a program that calculates the position of a planet for any date and time. If your instructor has given you a specific date and time, enter them on the dialog that appears, otherwise use the default (current date and local time) values. Press "OK" to compute the ephemeris, which will appear as a table on the screen. You will want to record the distance between the earth and Mercury (AU) and the expected length of time for the pulse to return. Leave the ephemeris window on the screen or minimize it (do not close it), and press "Set Coordinates". Respond "Yes" to "Use Computed Values?" and the telescope will begin slewing (moving rapidly) to point to the planet. (If you left the Ephemeris Computation window on the screen it will minimize when the slew starts. If you minimized it, it remains minimized and you can reopen it for reference.)

When the telescope move is complete, the red "Slewing" light will go out, a "Slewing Completed" message will appear on the display screen, and the telescope's pointing coordinates will appear on the control panel. (If you now turn off the drive by pressing the "Tracking" button you will notice that the right ascension increases as the Earth turns. Be sure to turn the tracking back on right away, so that the telescope doesn't lose its aim at the planet. You can always repeat the steps in the previous paragraph if you want to make sure you're pointed at Mercury, however.) When you are satisfied that the telescope is aimed correctly, press the "Send Pulse" button to transmit a radar pulse towards the planet. A "Pulse Sent" message will appear on the screen, along with the estimated time until reception of the return pulse. The message appears in yellow when the reflected pulse is returning, and the time appears in red during the last 30 seconds until reception.

A window showing the initial spectrum (frequency versus intensity) of the pulse also appears on the screen when the pulse is transmitted. This window will minimize itself after 10 seconds. When that occurs you will see an animated display on the telescope screen. This display shows the correct positions of Mercury, Venus, the Earth, and the Sun on the date of the Mercury coordinates you have used. Sketch them for further reference. The bright radar pulse moves toward Mercury and then the fainter echo returns. (The pulses move at the speed of light relative to the scale of the display. The distances on the display are correctly scaled, but the images of the disks of Sun, Earth, and Mercury are much too large for this scale.)

If for any reason you wish to stop waiting for a return pulse, press " Send Pulse" again, and respond "No" to the message that appears. While you are waiting for the return pulse, you can calculate d, x, and y for each of the time intervals (120, 210, 300, and 390 x 10^{-6} seconds), and write them down on the data sheet included in the lab write-up. (Refer to the *Sky and Telescope* reprint and to the later section of this write-up "Calculating the Rotational Velocity of Mercury from the Data" for instructions on how to calculate d, x, and y).



Taking Data

The return pulse is spread out over a few hundred microseconds in time, due to the curved surface of the planet. A series of five windows will appear on your screen when the pulse is received. These windows show "snapshots" of the spectrum of the returning echo beginning at the instant of reception, followed by another 120 microseconds later, and three more at successive 90 microsecond intervals. These pulse spectra show a certain amount of noise, or static, which increases with the later-arriving pulses, since they are weaker. Compare the appearance of the received pulse with the initial pulse. You will note that the initial pulse, which is of course stronger, appears much smoother and sharper.

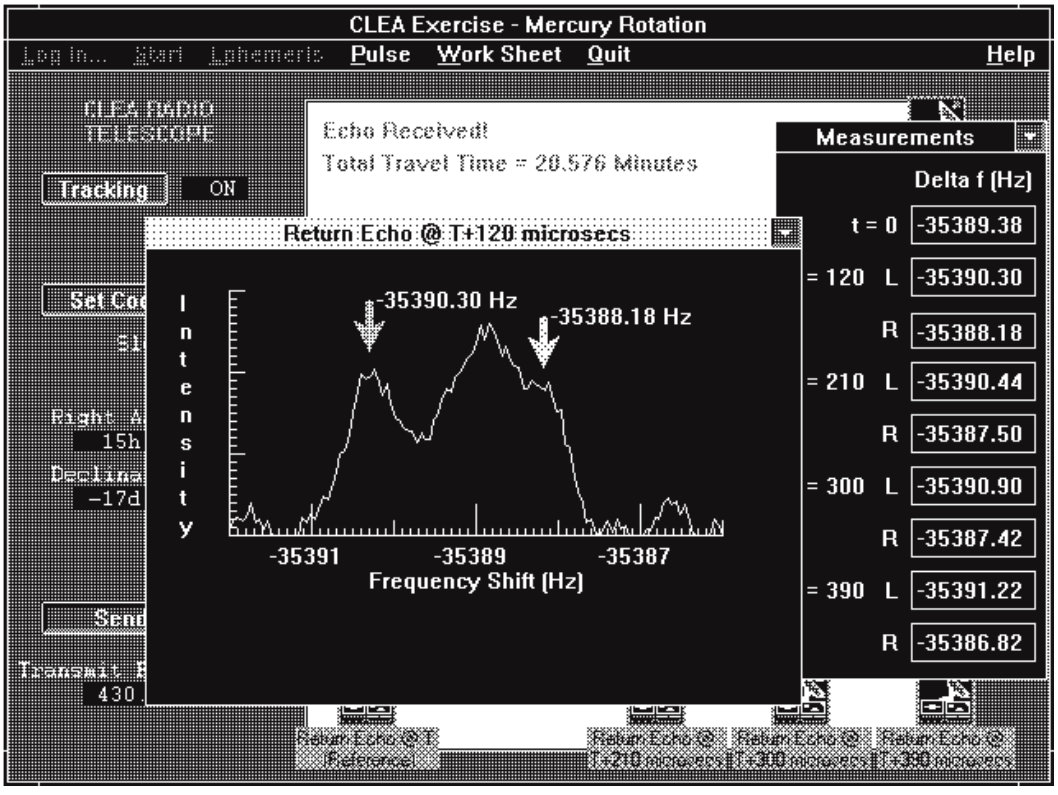
Clicking the cursor on any visible portion of a data window will bring that window to the "top" for observation and measurement. Clicking the down arrow in the upper left corner will reduce the window to an icon. An iconized window can be redisplayed by double clicking on it. A popup menu under "Pulse" on the main menu provides for reducing and displaying all the data windows at once, as well as recording measurements made on the pulse profiles. Data windows cannot be closed (unless a new radar pulse is transmitted), so that they can always be remeasured, if necessary.

To make a measurement, simply press and hold the left mouse button and move the cursor to the point to be measured. When the button is down the cursor becomes a cross-hair, and the lateral position of the mouse appears at the bottom of the screen as an X coordinate (measured in pixels), and a frequency shift in Hz from the zero position. For the sub-radar echo ($\Delta T = 0$) you simply measure the central frequency.

But for all the other "delayed" echoes, you must measure the positions of the left and right "shoulders" of the plots, the frequencies where the intensity just begins to fall off to zero. These represent echoes from the parts of the planet that are turning towards you and away from you the fastest. On some of the spectrum plots these points are quite obvious and on others they are not.

When you have positioned the vertical cross-hair on the left "shoulder" of the plot, double click the left mouse button. A red arrow will appear on the screen, along with the measured position in Hz. To measure the right "shoulder", follow the same procedure, but double click the right mouse button. A blue arrow will then appear at the measured position. (The single pulse on the sub-radar, t = 0, window may be measured by double clicking either mouse button.) If you want to change a measurement simply reposition the mouse and again double click the appropriate mouse button. The associated arrow will move to the new location. (It's possible to erase sections of the graph by repositioning the arrows. To restore the plot, iconize the window and then redisplay it.) You will want to write down the results of all of these measurements on the data sheet included in this write-up. (It's always good scientific practice to keep paper as well as computer records, if possible.)

After you have measured a pulse window (or all of them), select "Record Measurements" under "Pulse" on the main menu. A data window will appear recording all the measurements you have made. If you have not measured all the pulses, the remaining fields on the measurement window will be blank, and the next unmeasured pulse window will appear on top. If you have completed all the measurements, a message will inform you of this, and the pulse windows will be minimized when you respond to the message.



Recording and Printing Data

Once you have recorded the measurements of a pulse window (except the t=0 window), the "Work Sheet" selection on the main menu is activated. Select "Work Sheet" and then "Display", and the worksheet window is displayed. This worksheet is the same as the data table appearing on the back of your Laboratory Exercise, and you should fill in the data table with pen or pencil before you transfer the numbers to the computer. The computer only records raw data. It does not do the numerical calculations for you and it is up to you to make sure that the calculations are correct and are entered in their appropriate locations.

Follow the directions in your exercise to compute and fill in the entries in each column of the work sheet. (Note the notation used to indicate powers of ten in the work sheet entries. "Ten to the power" is indicated by "E" so that 5.6×10^{-8} would be written as 5.6E-08. If this is unfamiliar to you ask your instructor to explain it.) As you fill in each column, you may at any time press the "Check" button at the end of the column. The values you have entered will be checked for "reasonableness", and you will be informed if any of your values are clearly in error. You can go back to the pulse windows to make new or repeat measurements at any time Simply close the worksheet ("Work Sheet".."Close"), and then select "Pulse". Remember to record your measurements before returning to the work sheet.

If for some reason you want to completely recalculate a column, you can erase it. Select "Work Sheet".."Erase".."Column" (or "All") from the main menu. This will also erase the measurements.

To re-enter the measurements, select “Work Sheet”.."Close", then select "Pulse".."Record Measurements", then back to "Work Sheet".. "Display". Keep in mind that "Work Sheet".."Close" does not destroy any table entries, but "Work Sheet".."Erase" does.

When you have completed all entries in the work sheet and checked them for reasonableness, select "Work Sheet".."Print" for a hard copy of the table to turn in with your exercise.

CALCULATING THE ROTATIONAL VELOCITY OF MERCURY FROM THE DATA

You will want to use the following formulas to carry out the steps in transforming your raw data into values of the rotation period of Mercury. Make sure you have converted all measurements to the proper units as you go along. The first three items are geometrical terms needed to convert the measured velocities, which are from points not on the equator of Mercury, into velocities at the equator of Mercury. Since you know the anticipated time delays of the four echoes (120, 210, 300, and 390 μsec) you can calculate these items, d, x, and y, even before you have received the echoes. The other calculations are made with data measured from the spectrum of the echoes.

Item 1: Calculating d (in meters). This is the distance the delayed beam has travelled beyond the sub-radar point. We simply use distance = rate x time, but since we are measuring an echo, which has to travel over the same path twice (down and back) we take half this value.

$$\mathbf{d = (c\Delta t)/2}$$

Where c is the speed of light in meters per second (3×10^8 meters/sec) and Δt is the time delay for the particular pulse in seconds. (Note that one microsecond = 1 μsec = 10^{-6} seconds).

Item 2: Calculating x. This is the distance parallel to our line of sight from the center of Mercury to the point from which the echo comes back. It is just the radius of Mercury minus the distance d calculated in item 1, where the radius of Mercury is $R_{merc} = 2.42 \times 10^6$ meters.

$$\mathbf{x = R_{merc} - d}$$

Note that since the echoes we measure come back from points not far back (only a few kilometers) from the sub-radar point, x will be only slightly smaller than R.

Item 3: Calculating y. This is the distance perpendicular to our line of sight to the extreme outer edge of the region of Mercury from which the echo comes back. It is found by noting that y is one side of a right triangle whose hypotenuse is the radius of Mercury, and whose other side is x.

$$\mathbf{y = \sqrt{\left(R_{merc}{}^2 - x^2\right)}}$$

Item 4: Calculating Δf_{total}. This is the shift in frequency due to the rotational velocity alone. You need simply note that one side of Mercury that is rotating toward you as fast as the other side is rotating away. So the difference in the frequency shifts from the two extremes edges, Δf_{right} and Δf_{left} is twice the shift due to rotational velocity.

$$\mathbf{\Delta f_{total} = \left(\Delta f_{right} - \Delta f_{left}\right)/2}$$

Item 5: Calculating Δf_c. This is the shift in frequency corrected for the fact that this is an echo—the shift is twice that produced by a source which is simply emitting at a known frequency. This is because the pulse arriving at Mercury appears shifted as seen from the surface, and then it is shifted again because the surface of Mercury is moving as seen from the earth.

$$\mathbf{\Delta f_c = \Delta f_{total}/2}$$

Item 6: Calculating V_o. This is the component of the rotational velocity of the edge of Mercury along the line of sight at the point from which the echo returns. We simply apply the Doppler equation to the observed frequency shift.

$$\mathbf{V_o = c\left(\Delta f_c/f\right)}$$

Where, as before, c is the speed of light in meters per second, and f is the unshifted transmitted frequency of the pulse (which can be read from the control panel of the radio telescope, but note that f must be in Hertz, Hz, not Megahertz. On the telescope display 1 MHz = 10^6 Hz.)

Item 7: Calculating V. This is the equatorial rotational velocity of the planet Mercury, and is just your measured velocity times a geometrical factor, R_{merc}/y which corrects for the fact that the velocity you measure is only the component of the rotational velocity directed along your line of sight, and that the component perpendicular to the line of sight produces no measurable Doppler shift.

$$\mathbf{V = V_o\left(R_{merc}/y\right)}$$

Item 8: Calculating P_{rot}. For each of the delayed echoes you can now calculate a rotational period for the planet by dividing the circumference of Mercury, $2\pi R_{merc}$, by its velocity , and dividing the result (which will be in seconds), by the number of seconds in a day: 86,400.

$$\mathbf{P_{rot}(seconds) = 2\pi R_{merc}/V}$$

FINAL RESULTS

When you have calculated P_{rot} for each of the delayed echoes, make sure your answers are reasonable. Then compute an average period of rotation in days for Mercury from your values.

The Rotation Period of Mercury = _____ days.

What is the percentage difference between this period and the accepted value of 59 days?

$$\frac{P_{rot} - 59}{59} \quad \text{x } 100\% = _____.$$

THE ORBITAL VELOCITY OF MERCURY

You can use your value of frequency shift for the echo from the sub-radar point to calculate the orbital velocity of Mercury. Just note that the shift you get must be divided by two to account for the doubling due to an echo. Apply the Doppler formula to the shift to calculate the speed of the planet. Negative speeds are speeds of recession, and positive speeds are speeds of approach. Show the data you use and the calculations below. Express your answer in km/sec (1 km = 10^3 meters)

Orbital Velocity of Mercury: _____

Sketch the relative positions of earth, sun, and mercury, along with the direction of Mercury's orbit. Does your answer for the orbital velocity seem reasonable? Explain.

EXTRA-CREDIT PROBLEM

The relative sizes of the orbits of the planets were known (from Kepler's laws) long before the actual number of kilometers in an astronomical unit was measured. The delay in the return time of a radar signal provides a neat and accurate way of measuring the astronomical unit (AU). Your ephemeris calculations gave you the distance of Mercury from the earth in AU. Use this value, and the measured time for the two-way travel of the pulse to calculate the number of kilometers in an AU. For this purpose, note that c, the speed of light, is 2.998×10^5 kilometers/second. Show your work carefully below.

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Mercury Lab Data Table

delta t (microsecs)	120	210	300	390
d (meters)				
x (meters)				
y (meters)				
freq → left (Hz)				
freq → right (Hz)				
delta f_{total} (Hz)				
delta f_c (Hz)				
V_0 (meters/sec)				
V (meters/sec)				
P_{Rot} (days)				

DELTA f
for Subearth
pulse (T=0) _____

$V_{orbital}$ _____