THE HUBBLE REDSHIFT DISTANCE RELATION

SOFTWARE USERS' GUIDE

A Manual to Accompany Software for
Introductory Astronomy Lab Exercise
Document SUG 3: Version 1

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I. INTRODUCTION

I-1. Purpose

This document provides specific technical information for users of the CLEA software module **"The Hubble Redshift - Distance Relation"**. It includes discussions of hardware and operating system requirements (Section II); software installation and operation (Section III); files associated with the exercise (Section IV); user options and file formats for possible user modifications (Section V); and astronomical data, models and algorithms employed in the exercise (Section VI). The material in this guide is intended for use by the instructor and by other personnel involved in installing and configuring the exercise software and PC hardware. It is not intended for distribution to the students. (The word "user" when it appears herein should be interpreted as "instructor", and not "student".)

Readers of this document are assumed to have available the **General Users' Guide to CLEA Windows Software**, which contains technical information that applies to all CLEA Windows software. Information contained therein is not repeated here.

I-2. Acknowledgements

Development of CLEA exercises and materials, including this guide, is supported in part by National Science Foundation Grant USE 9155927 and by Gettysburg College.

II. SYSTEM REQUIREMENTS

This software runs on IBM and compatible PCs under Microsoft Windows 3.0/3.1. It requires a hard disk with at least 1.5 MByte of available storage, and a color VGA monitor. For additional information on systems requirements, including a discussion of color modes and video drivers, see the **General Users' Guide to CLEA Windows Software**.

III. INSTALLATION AND OPERATION

All files for this exercise are compacted into the distribution file CLEA_HUB.ZIP. They must be extracted using the PKUNZIP program (also included on the distribution diskette), and placed together in a single directory, such as C:\HUB_LAB. Details of unpacking, running, and installing this software under Windows are given in the **General Users' Guide to CLEA Windows Software.**

IV. FILES INCLUDED WITH EXERCISE

The following files comprise this exercise, and should be present in your working directory after unpacking the distribution (.ZIP) file:

README.HUB - Notes on the current version, including any incompatibilities with previous versions, old files that should be deleted, etc.

UPDATE.HUB - A list of updates (by version) since version 0.50.

USRGUIDE.HUB - This document, in text form.

CLEA_HUB.EXE - The executable code for the exercise.

GENSTARS.EXE - Executable code for the GENSTARS program (see Appendix).

CLEALOGO.BMP - 256-color bitmap data for the CLEA Logo screen.

CLEALG16.BMP - 16-color bitmap data for the CLEA Logo screen.

HUBLAB.BMP - 256-color bitmap data for the Title screen.

HUBL16.BMP - A 16-color version of the Title screen.

HUBFILES.DAT - A list of files containing star fields and the field coordinates (see Section V-3.1).

BOOTFLD.DAT

COMAFLD.DAT

CRBORFLD.DAT

UMAFLD1.DAT

UMAFLD2.DAT - Positional, photometric, and other data for artificially generated star/galaxy fields (see Section V-3.2).

GLIMAGES.DAT - A list of the galaxy image bitmap files, including dimensions and image centers (see Section V-3.3).

GLX10.BMP
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GLX150.BMP - Bitmap data for 15 galaxy images (see Section V-2).

GLY10.BMP
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GLY150.BMP - Bitmap data for reduced images of the same 15 galaxies (see Section V-2).

CLEAHELP.HLP - Text for "Help"..."On Help".

HUBSTART.HLP - Text for "Help"..."Getting Started".

HUBDATA.HLP - Text for "Help"..."Taking Data".

REMINDER.HUB - Text to display when "Quit" selected.

(HUBOPTS.INI) - **NOTE:** This file is not included in the distribution, but is created the first time the exercise is run. It contains default settings for file names and options. This file can be deleted if desired, it will then be re-created.

V. USER OPTIONS

In this exercise, the instructor can replace/add galaxy images, modify/add star-galaxy fields, and modify/add help screens. Further options allow the user to set the Hubble parameter to a preferred value, adjust magnitude and other parameters affecting the generation and display of the spectra, and set the mean absolute magnitude of a galaxy. This adaptability is provided through two mechanisms: an options dialog that enables the user to select built-in options and change file names and path; and the files themselves, which in many cases can be modified or replaced. The parameters and selectable options for this exercise are presented in Section V-1, the galaxy image bitmaps are described in Section V-2, and the formats and contents of the various data files (*.dat) are discussed in Section V-3. See the **General Users' Guide to CLEA Windows Software** for a general discussion of user options, including password access to the options dialog, as well as bitmap and help files.

V-1. Parameters and Selectable Options

The following parameters are set via the options dialog:

UT-Local Time (Hrs): The value (in integer hours) of Universal Time minus local (zone) time for the presumed location of the "observatory" for this exercise. This number is used to convert the time obtained from the system clock on the individual PC to Universal Time. The table below gives the standard time values for North American time zones. (For daylight time, subtract one from the table value.) The default is 5 hours (Eastern Standard Time).

Zone	UT-ST (Hrs)
Atlantic	4
Eastern	5
Central	6
Mountain	7
Pacific	8
Alaska	9
Hawaii	10

H (Km/Sec/MPc): This option allows the instructor to adjust the exercise to yield whatever value of the Hubble parameter is preferred, since the issue is still somewhat in question. The apparent magnitudes of the galaxies are computed using this parameter, the absolute magnitude (below), and the individual redshifts (Section V-3.2). The default value of H is 75.0 (Hodge, 1993).

Galaxy Absolute Mag: Via this parameter, the instructor may also adjust the absolute magnitude assigned to the galaxies. (There is no current mechanism for assigning different absolute magnitudes to different galaxies.) The default value is -22.0.

"Noise" in Abs Mag: The amplitude of the “random” (see Section VI-3) noise applied when computing the apparent magnitude of a galaxy, so that all galaxies in a cluster do not have the same apparent magnitude. The default value is 0.5.

Base Magnitude: The magnitude used as a base for the spectrum integration calculations (see Section VI-3). This parameter should be set to a value equal to or brighter than the apparent magnitude of the brightest galaxy, and adjusted so that the integration times for the faintest galaxies are not unduly long (perhaps 5 minutes to an SNR of 7 or better). The default value is 12.5.

Sky Magnitude: The magnitude of the sky, used in the spectrum integration calculations (Section VI-3). Setting this parameter brighter will increase the sky background and decrease the strength of the lines in the spectra of fainter galaxies. The default value is 19.0.

Display Update Freq: The number of photon counting loops performed between updates of the spectrum display. (See discussion in Section VI-3.) The default value is 20.

Table Number: This field allows the instructor to preset the table number for each machine in the lab. This number then appears as the default on the login dialog. The table number has a maximum length of 4 characters, but need not be numeric.

Options Password: This is the password for access to the options dialog. Allowable passwords have a maximum length of 16 characters, and are case sensitive. The default password is "CLEA".

V-2. Galaxy Image Bitmaps

The galaxy images are monochromatic bitmaps, extracted from image files of fields of galaxies and converted to .BMP format. The image dimensions are not fixed, but should be made as small as possible to contain the image. A second, reduced image of the galaxy (1/3) must be created, most image processing software will do this. (The results are generally much more satisfactory when a larger image is reduced than when a smaller one is enlarged.) The width and height of each image (in pixels), should be recorded, along with the coordinates (X,Y in pixels) of the center of the galaxy image (not the center of the bitmap (i.e. width/2, height/2)). This information, along with the name of the .BMP file and an index number, must be entered in the galaxy image list file (Section V-3.3). Assigned index numbers must be unique within the list, and the reduced images must carry an index one greater than the index of the associated larger image.

Although the images are monochromatic, they require a 256-color graphics mode (See the **General Users' Guide to CLEA Windows Software**, Section II-4) to provide sufficient levels of gray scale. Alternate 16-color bitmaps of the galaxies are not provided. The results of reducing the images to 16 colors was found to be essentially the same as when the supplied versions are displayed in 16 colors. (The standard 16 colors include only a few levels of gray scale - black, dark gray, gray, light gray, white.)

V-3. Data (.DAT) Files

Data for the star fields and individual stars used in this exercise is read from files typically identified by the qualifier .DAT. These files, like the .HLP files (Section V-3), are in text format, and can be created and maintained with a text editor. (See remarks in Section V-3 concerning the use of word processors.) The specific record formats for the star fields list file, the field data files, and the galaxy image list file are given in Sections V-3.1, V-3.2 and V-3.3, respectively. The following general rules apply:

1) Data should be entered one record per line.

2) Fields within records are separated by at least one space. Do not use commas or other separators, and do not embed spaces in numeric fields.

3) String fields are enclosed in single quotes ('), and the number of characters + spaces between the quotes must be <= the maximum length of the field.

4) Real fields may carry a sign, and may be in decimal (1.0, -2.345) or exponential (0.1234E-06, -8.549E+18) form. Real fields will also accept integer (0, 1, -99) values.

5) Integer fields may carry a sign, and cannot include a decimal point or exponent.

6) Boolean fields may be assigned only the values 0 (false) and 1 (true).

7) The maximum allowed values for all numeric fields (in terms of both absolute value and number of digits) are much larger than required for any reasonable values for the quantities they represent.

8) Values must be provided for all fields in a record, in the proper order. (Exception: all remaining empty fields on the end of records may be omitted.) For null strings enter two single quotes (' '), for null numeric fields enter 0 or 0.0.

9) There is no specific limit on the number of records in a file. However, these files are read into and maintained in memory when they are used, so there is some risk of running out of heap space with a large file.

To add a new star field to the exercise, you must create a stellar data file and add galaxy images (Section V-3.2), place it in the exercise directory (Section III), and add a record to HUBFILES.DAT (Section V-3.1) naming the file and specifying its coordinates. Before creating new fields or adding data to existing fields you should read Section V-3.4, which presents some restrictions and guidelines on star field coordinates and sizes. For any new galaxy images you have added you must supply two .BMP files (Section V-2) and add records to GLIMAGES.DAT (Section 4-3) identifying the image bitmaps.

V-3.1. Star Fields List File

This file provides a list of the files that contain the star and galaxy data for each star field, along with the coordinates of he field centers. It is identified in the field labeled "Star Fields List" on the options dialog (Section V-1); the default is HUBFILES.DAT. Records in this file have the following format see general rules under V-3): When the telescope is moved via the "Set Coordinates" button, the program searches the star fields list to find the field center closest to the target coordinates, and loads the corresponding field data file.

Field	Data Type	Contents
File Name	String[12]	Name of field data file (name + qualifier, no path).
Right Ascension	Real	RA of field center in decimal hours (0.0..24.0, but see Section V-3.4).
Declination	Real	Dec of field center in decimal degrees (-90.0..+90.0, but see Section V-3.4).
Field Name	String[20]	Field identification for selection lists.
Default	Boolean	One record in the file may carry a 1 in this field to identify the default star field.

NOTE: The star fields list is not searched during a slew (using the N-S-E-W buttons). It is therefore possible to slew off a star field into empty space, even though a neighboring field may be available. Always use the "Set Coordinates" button (or the "Change Field" menu selection) to change fields. (Of course, if the nearest field center is too far away or the field is small you may still get an empty screen when using the "Set Coordinates" button.)

V-3.2. Field Data Files

These files contain positional and other data for each star and galaxy in a star field. Each entry in the star fields list (Section V-3.1) identifies a field data file, which must be present in the working directory when the exercise is run. (Note hat these files are not identified on the options dialog (Section V-1).) Five of these files are supplied with the exercise. The user may add as many as desired (supplying corresponding entries in HUBFILES.DAT), and may also add/delete/modify data in the files provided. Field data file records have the following format (see general rules under V-4):

Field	Data Type	Contents
Star/Galaxy ID	String[15]	Optional name, HD/BD number, Messier/NGC number, or other identification. *Note 1*
Right Ascension	Real	RA of object in decimal hours (0.0..24.0).
Declination	Real	Dec of object in decimal degrees (-90.0..+90.0).
Magnitude	Real	Apparent magnitude of object (stars), or unused (galaxies)
Z	Real	Red shift (=Delta_Lamda/Lamda) (galaxies).
(Unused)	Real	For future use.
(Unused)	Real	For future use.
(Unused)	Real	For future use.
(Unused)	Byte	For future use
(Unused)	Byte	For future use
(Unused)	Byte	For future use.
Object Code	Byte	Identifies object as a star (0), or galaxy (1).
Image Index	Integer	Index number of associated "large" galaxy image bitmap in galaxy image list file (Section V-3.3) (galaxies).
Comment	String[23]	Optional additional information on object *Note 1*.

Notes:

(1) Enter two single quotes (' ') to blank out this field.

V-3.3. Galaxy Image List File

This file provides a list of the bitmap (.BMP) files containing the galaxy images. Note that both"“small" and "large" (magnification factor 3) images must be provided for each galaxy (Section V-2).

Field	Data Type	Contents
Index	Word	Unique index number to identify the image. The assigned value is more or less arbitrary, but "small" images must carry an index equal to the index of the associated "large" image + 1.
File Name	String[12]	Name of bitmap file containing the image (name + qualifier (.BMP), no path).
Image Width	Byte	Width of the image, in pixels.
Image Height	Byte	Height of the image, in pixels,
X Center	Byte	X coordinate (pixels) of the center of the galactic image (not the center of the bitmap).
Y Center	Byte	Y coordinate (pixels) of the center of the galactic image (not the center of the bitmap).

V-3.4. Field Coordinates and Angular Size

When creating new star fields for this exercise, or modifying existing fields, give consideration to the following:

1) The "finder" (wide) field of the simulated telescope is 2.5 degrees square. Users may want to provide data for a larger field to avoid having part of the visible field go blank when slewing to objects of interest.

2) The objects of interest (galaxies) should be placed near the center of the field. The coordinates of the field center should be entered in the star field list file (Section V-3.1).

3) Avoid high declinations (stay within -60 < dec < +60). The display algorithm is relatively simple, similar to a Mercator projection.

4) Similarly, avoid fields that straddle the 24Hr/0Hr dichotomy in right ascension. (The program will not identify a coordinate of, for example, 23Hr 50Min RA as being "near" one at 0Hr 5Min, etc.)

We have developed a separate program, GENSTARS, which can be used to generate random star fields, primarily for backgrounds. The GENSTARS program is described in the Appendix.

VI. ASTRONOMICAL DATA, MODELS AND ALGORITHMS

VI-1. Star Images

A frequent problem with much astronomical software we have seen, even some that is otherwise quite sophisticated, is that the plotted star fields do not look realistic. The most common echniques are to either plot all stars as a single pixel of the same intensity, or to use filled circles of varying sizes to represent the magnitudes. The problem with the first method is that while the images look somewhat like stars, the field does not look like the sky (picking out constellations or other patterns is difficult), and attempting to vary the intensity of a single pixel yields a useful range of only two or three "magnitudes". On the other hand, the second method yields images that look nothing at all like stars, with the brighter "stars" being large uniform sharp-edged circles, some as large as dimes. As we feel that it is important for the students to appreciate the true appearance of the sky, both to the naked eye and through various astronomical instruments, we devoted some time to the creation of more realistic looking star images, for this and for other CLEA exercises.

Clearly, the ideal way to display a realistic star image is to use a real star image, which will not be smooth and perfectly round, but will have natural irregularities caused by seeing, optics, etc. A CCD image is the obvious choice, since it is already in digital form. We began by selecting a single fairly bright CCD star image. We then developed a series of successively fainter images by reducing the intensities, pixel by pixel, in equal stages. The factor used to decrease the intensity at each stage was experimentally adjusted, balancing the desire for a reasonably wide magnitude range against the need for each image to be "clearly fainter" in appearance than its successor. (The faint end of this process was reached when the resulting image was barely visible on the screen against a black background.) This eventually resulted in a series of 11 images. The brightest image was (more or less arbitrarily) assigned magnitude 0, and each succeeding image increased by 1 magnitude to give a base magnitude range of 0 through 10. When stars are plotted, their magnitude is rounded to a whole number and the corresponding image is assigned. (A bias can be added to map the stellar magnitudes to the range of images.) Stars fainter than the end of the image range are all plotted using the faintest image.

We have not attempted to add color to the star images. The images are displayed on the screen by setting all three color values (RGB) to the same level. Intensities in the image data that exceed 256 (in the original (Mag 0) image the values go as high as 17,000) are set to 256, the highest available. This yields a burned-out center, which is realistic for TV monitors. When the images are displayed in a 16-color video mode the edges of the brighter images show some step contouring, due to the limited number of gray scale values available.

In order to provide a method of quickly generating star fields for exercises such as this, we have developed program GENSTARS. This software generates a file of data records (Section V-3.2) representing a random field of stars, as specified by parameters supplied by the user. The GENSTARS program is described in the Appendix and is supplied as part of this exercise. All of the star fields provided with this exercise were created with GENSTARS, and therefore do not accurately depict the sky at the given coordinates.

VI-2. Galaxy Images

The galaxy images were taken from available collections of astronomical images on CD ROM. They were selected largely on appearance and prepared as described in Section V-2. As is the case with the star fields, the images do not match actual galaxies in the clusters listed, although clusters at greater distances were given successively smaller images. The galaxy images were added to the generated star fields by editing the files and supplying the appropriate records (Section V-3.2).

VI-3. Observed Quantities

The redshifts assigned to each galaxy are based on the average value for the cluster, from Zombeck (1990). In order to allow the instructor to set the desired value of the Hubble parameter H (Section V-1), the apparent magnitude for each galaxy is computed from the given value of H, the galaxy's redshift Z (Section V- 3.2), and the average absolute magnitude for a galaxy (also an adjustable parameter - Section V-1). (We plan to include multiple absolute magnitudes, based on the galaxy's morphological type, in a future version of this exercise.) The assigned apparent magnitude includes a random noise component, which is computed using the "Noise" in Absolute Magnitude parameter (Section V-1). Due to this method of assigning magnitudes, the faintest appearing galaxy in each field may not necessarily be assigned the faintest apparent magnitude, although the differences are not great. In order to assure that a particular galaxy will always be assigned the same apparent magnitude, a random number seed is generated from the galaxy's Right Ascension. (While it might be of some value to have students at different tables use different values, it is clearly unacceptable to assign a different apparent magnitude each time the field is loaded, such as in returning to a field to repeat measurements.)

The program adjusts the integration rate of the spectra based on both the apparent magnitude of the galaxy and the surface brightness within the slit. This feature was added primarily to realistically adjust for poor image centering, but also results in longer integration times for galaxies of nearly the same apparent magnitude that have lower surface brightness. (The program makes no other adjustments for the position of a galaxy in the slit. Positioning the slit on a spiral arm, for example, will not yield rotational velocities or a change in spectral appearance caused by a greater number of early type stars.)

The continuum radiation in the galactic spectra is taken to be that of a 5800 K black body. The effective temperature is modified by the red shift, as are the positions of the spectral lines (the H & K lines of CaII). The applicable formulae and values are taken from Lang (1980). The appropriate black body spectrum is generated, the H & K lines are added at their redshifted positions, and an array of normalized data points (Spect(I)) is generated, covering the observed wavelength range 3900 to 4900 Angstroms.

In order to simulate the Poisson statistics of the photons, a probability P(I) = (B(galaxy)/B(base)) * Spect(I) is assigned to each "channel" of the photometer. The ratio B(gal)/B(base) is determined from the magnitude equation:

$$m(gal) - m(base) = -2.5*\log(B(gal)/B(base))$$

Here m(base) is the Base Magnitude parameter (Section V-1). A similar ratio B(sky)/B(base) is obtained from the Sky Magnitude parameter, and its value is assigned to the probability P(Sky).
The counts S(I) are then built up by continually looping through the S array, performing the following for each point:

```
if (Random <= P(I)) then Increment(S(I))
if (Random <= P(Sky)) then Increment(S(I))
```

The running totals S(Signal) and S(Sky) are also maintained separately and used to compute the signal to noise ratio:

```
SNR = (S(Signal)/Channels) / sqrt((S(Signal)+S(Sky))/Channels)
```

Clearly, if the Base Magnitude is set fainter than a galaxy's apparent magnitude, the resulting P(I)s may be greater than 1. In this case we simply assign 1.0 to the ratio B(gal)/B(base), so that the spectra for all galaxies brighter than the Base Magnitude will integrate at the same rate.

A drawback to this technique is that the rate of integration is ultimately determined by the speed of the computer. A significantly greater factor than the time required by the processor to perform the instructions in each loop is the time required to normalize the current counts and display them on the screen, which can show a great variation from machine to machine. For this reason we have included the Display Update Frequency parameter (Section V-1). This parameter is the number of times that a complete loop of the S array is performed before the results are normalized and displayed on the screen. Setting this parameter to a higher value can significantly speed up spectrum integration on slower machines. On the other hand, frequent screen updates on machines fast enough to perform them without bogging down can provide an educational display of statistics in action.

The default values for the parameters Base Magnitude (12.5), Sky Magnitude (19.0), and Display Update Frequency (20), have been found to give satisfactory results on a 25 MHz 386SX machine with a 387 coprocessor. On a faster machine (486DX2/50 with an ATI ULTRA graphics card), setting the parameters to (respectively) 10.5, 17.0, and 10 gave useful and educationally valuable results.

At the present time, positioning the slit on a star image will yield only the sky background (P(I) = 0). We plan eventually to add stellar spectra and develop exercises in spectral classification and radial velocities.

APPENDIX

THE GENSTARS PROGRAM

GENSTARS is a separate Windows program, developed by CLEA, that generates random star fields for use with the Hubble Expansion exercise and other applications. GENSTARS.EXE is included on the distribution diskette as part of the Hubble Expansion exercise. It can be installed under Windows by following the steps given in Section III-2.2, with the following changes under Step 5:

Description: "GenStars"
Command Line: "C:\HUB_LAB\GENSTARS.EXE"

Each generation performed under GENSTARS produces a field data file (Section V-3.2) containing star data records. The fields in the generated records are filled as follows:

Field	Data Type	Contents
Star ID	String[15]	A running number starting with 1, in string format.
Right Ascension	Real	A randomly assigned right ascension, between user-specified limits.
Declination	Real	A randomly assigned declination, between user-specified limits.
Magnitude	Real	A randomly assigned apparent magnitude, between user-specified limits. An exponential "luminosity function" is used to increase the numbers of stars at fainter magnitudes.

All other numeric fields in the record are zero (0 or 0.0). The comment field is given a null string (' '). A text editor may be used to modify the generated records. For the Hubble exercise, records specifying the galaxy images may be added, or generated records may be altered. (The Galaxy ID, Z, Object Code, and Image Index fields must be assigned values - see Section V-3.2).

GENSTARS is simple and straightforward to use. Begin by selecting "Field Parameters" on the main menu. A dialog box appears with entries specifying the right ascension, declination, and magnitude ranges, the output file name (which may include a path - up to 80 total characters), the number of stars, and the random seed. The generations are repeatable if the same random seed is used

When you have finished entering the parameters, press "Ok". The field parameters will then be displayed in the main window. You may still change them by selecting "Field Parameters" again. If you are satisfied with the parameters, select "Generate Field" from the main menu. The generation is performed quite quickly and "*** Generation Completed ***" will appear under the parameters in the main window. (If you are overwriting an existing file, a message will warn you of this and allow you to cancel or continue.)

When the generation is complete, the "Generated Data" item on the main menu is activated. Selecting this enables you to display he new star field ("View"), or view/modify the generated file records via the Windows Notepad ("Edit").

You may return to "Field Parameters" and perform a new generation as many times as you like, either naming a new file for the results or writing over the previous. (As noted earlier, the generations are repeatable if all the parameters - particularly the random seed - are the same.) Creating a star field that appears "authentic" for a particular application may require some experimentation, varying in particular the magnitude range and number of stars. When you have finished, select "Finished" from the main menu to end the program.

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