Bull. London Math. Soc. 37 (2005) 172–178
doi:10.1112/S0024609304004151

BRAUER GROUP INVARIANTS ASSOCIATED TO ORTHOGONAL EPSILON-CONSTANTS

DARREN GLASS

Abstract

In this paper, the theory of ε-constants associated to tame finite group actions on arithmetic surfaces is used to define a Brauer group invariant $\mu(\mathcal{X}, G, V)$ associated to certain symplectic motives of weight one. The relationship between this invariant and $w_2(\pi)$ (the Galois-theoretic invariant associated to tame covers of surfaces defined by Cassou-Noguès, Erez and Taylor) is also discussed.

1. *Introduction and background*

In his paper [**4**], Deligne used elements in the Brauer group of $\mathbb{Q}$ and their relationship with certain ε-constants to give a proof of the Fröhlich–Queyrut theorem. In particular, he showed that certain global orthogonal root numbers are equal to one, by interpreting the associated local orthogonal root numbers as Stiefel–Whitney classes and then using the local root numbers to define an element of order two in the Brauer group of $\mathbb{Q}$. This idea was furthered by Saito (in [**13**], for example) and others who defined Brauer group invariants associated to situations which can be interpreted as motives that are orthogonal and of even weight. In this paper, we define a Brauer group invariant associated to certain motives that are symplectic and have weight one.

In order to construct the relevant motives, we first define $\mathcal{X}$ to be an arithmetic surface of dimension two which is flat, regular, and projective over $\mathbb{Z}$. Throughout this paper, we assume that $f : \mathcal{X} \longrightarrow \operatorname{Spec}(\mathbb{Z})$ is the structure morphism. Let G be a finite group that acts tamely on $\mathcal{X}$. In other words, for each closed point $x \in X$, the order of the inertia group of x is relatively prime to the residue characteristic of x. Let $\mathcal{Y}$ be the quotient scheme $\mathcal{X}/G$, which we assume is regular, and assume that for all finite places v, the fiber $\mathcal{Y}_v = (\mathcal{X}_v)/G = \mathcal{Y} \otimes_{\mathbb{Z}} (\mathbb{Z}/p(v))$ has normal crossings and smooth irreducible components with multiplicities relatively prime to the residue characteristic of v. Finally, let V be a virtual representation of G over $\bar{\mathbb{Q}}$. In particular, V is a linear combination of representations.

We wish to define a class $\mu(\mathcal{X}, G, V)$ in the global Brauer group $H^2(\mathbb{Q}, \mathbb{Z}/2\mathbb{Z})$ that will depend only on $\mathcal{X}, G$, and V. In particular, $\mu(\mathcal{X}, G, V)$ will be the element whose local invariant at the place v is given by the sign of the local epsilon constant $\varepsilon_v(D', V)$, where D' is an appropriately chosen horizontal canonical divisor on $\mathcal{Y}$. For details on local epsilon constants, and in particular the fact that they can be defined independent of all choices, we refer the reader to [**3**].

Recall that a *horizontal canonical divisor* is a divisor D' such that $O_{\mathcal{Y}}(D' + \mathcal{Y}_T)$ is isomorphic to $\omega_{\mathcal{Y}/\mathbb{Z}}(\mathcal{Y}_S^{\mathrm{red}})$, where $\mathcal{Y}_T$ and $\mathcal{Y}_S$ are unions of vertical fibers of $\mathcal{Y}$.

Received 14 January 2003; revised 24 March 2004.

2000 *Mathematics Subject Classification* 11G35 (primary), 11G40, 14G40 (secondary).

This work received support from NSF grant DMS-98-10750.

In addition to the requirement that it be a canonical divisor, we must impose an additional condition on our choice of D' in order for $\mu(\mathcal{X}, G, V)$ to be well defined. To do this, we recall that [**7**, Proposition 3.10] shows that if F' and G' are components of $\mathcal{Y}_v^{\mathrm{red}}$, and D' is a canonical divisor in the above sense, then there is a canonical isomorphism between $\mathcal{O}_{F'}(D' \cap F')$ and $\omega_{F'}(F' \cap G')$. This isomorphism maps the global section $1 \in \Gamma(\mathcal{O}_{F'}(D' \cap F'))$ to an element $\gamma \in \Gamma(\omega_{F'}(F' \cap G'))$ such that γ has a simple pole at all $x \in F' \cap G'$. Define a_x to be the residue of γ at these points x. We wish to choose D' so that all of the residue terms a_x are equal to 1, and we will call such a choice of D' a *divisor with identity residues at crossings*. Such divisors exist due to a moving lemma that we make explicit in Section 2. We can now state the main theorem.

THEOREM 1.1. *In the case where V is an orthogonal virtual representation of dimension equal to zero and trivial determinant, and where D' is a divisor with identity residues at crossings, the local constant $\varepsilon_v(D', V)$ is independent of the choice of D'. In particular, the element $\mu(\mathcal{X}, G, V)$ in the global Brauer group $H^2(\mathbb{Q}, \mathbb{Z}/2\mathbb{Z})$ whose local invariant at the place v is given by the sign of $\varepsilon_v(D', V)$ is well defined.*

In Section 2 of this paper, we prove Theorem 1.1. In Section 3, we recall the definition of the Galois-theoretic invariant $w_2(\pi)$, defined by Cassou-Noguès, Erez and Taylor in [**1**], which comes from a situation similar to the one in which we are working, and we prove a connection between the two invariants. Finally, in Section 4 we prove the moving lemma that we use in Section 2 in order to show that divisors with identity residues at crossings always exist.

2. *Definition of* $\mu(\mathcal{X}, G, V)$

In [**7**], the author proves the following result regarding orthogonal ε-constants associated to tame finite group actions on surfaces. (We refer the reader to [**3**], [**2**] and [**7**] for all relevant definitions.)

THEOREM 2.1. *Let v be any finite place of $\mathbb{Q}$, and let $\mathcal{Y}_v$ be the fiber of an arithmetic surface $\mathcal{Y}$ over the place v. Let C_i be the components of $\mathcal{Y}_v$, and let Z be the set of crossing points of the C_i. Furthermore, choose D' to be a canonical horizontal divisor on $\mathcal{Y}$. If V is an orthogonal virtual representation of degree zero and trivial determinant, then we have the following formula:*

$$\frac{\varepsilon(\mathcal{Y}_v, V)}{\varepsilon(D'_v, V)} = \prod_i \det(V^{I_i})(\delta_{v,C_i}) \prod_{z \in Z} \varepsilon_{0,z}(C_{i_1}, V^{I_{i_1}}) \varepsilon_{0,z}(C_{i_2}, V^{I_{i_2}}) \varepsilon(z, V),$$

where the first product ranges over the components C_i, and I_i is the inertia group of the generic point of C_i.

In particular, we note that the terms δ_{v,C_i} live inside the generalized class groups

$$H^1(C_i \mod C_i \cap Z, K) = \left[\Big(\bigoplus_{x \notin C_i \cap Z} \mathbb{Z} \Big) \oplus \Big(\bigoplus_{x \in C_i \cap Z} K^*/U_x^1 \Big) \right] / K^*,$$

where K is the fraction field of C_i (see [**12**] and [**7**] for details).

By saying that D' is a *canonical divisor*, we mean that D' is a horizontal divisor on $\mathcal{Y}$ such that $D' + \mathcal{Y}_T = K_{\mathcal{Y}} + \mathcal{Y}_S^{\text{red}}$, where $\mathcal{Y}_S^{\text{red}}$ is the sum of the reductions of the fibers of $\mathcal{Y}$ at the places in the set S of bad primes, and $\mathcal{Y}_T$ is the sum of the (necessarily reduced) fibers of $\mathcal{Y}$ over the places in a set T that is disjoint from S. In particular, this will imply that $O_{\mathcal{Y}}(D' + \mathcal{Y}_T)$ is isomorphic to the twist $\omega_{\mathcal{Y}/\mathbb{Z}}(\mathcal{Y}_S^{\text{red}})$ of the relative dualizing sheaf $\omega_{\mathcal{Y}/\mathbb{Z}}$ by $\mathcal{O}_{\mathcal{Y}}(\mathcal{Y}_S^{\text{red}})$. We also wish to choose D' so that it intersects the non-smooth fibers $\mathcal{Y}_v$ of $\mathcal{Y}$ transversally at smooth points on the reduction of $\mathcal{Y}_v$.

Given any such horizontal divisor D', it is possible to find a divisor with identity residues at crossings that is close to it, due to the following moving lemma, which is proved in Section 4. In particular, this shows that divisors D' satisfying our hypotheses always exist, and therefore that our class $\mu(\mathcal{X}, G, V)$ will be well defined.

LEMMA 2.2. *There exists a meromorphic function h on $\mathcal{Y}$ such that the divisor of h intersects the special fibers $\mathcal{Y}_v^{\text{red}}$ transversally at smooth points away from D'_v, and such that h takes on prescribed values at the singular points of $\mathcal{Y}_v^{\text{red}}$. In particular, given a horizontal divisor D' as in the previous section, the divisor $D' + \operatorname{div}(h)$ will have residue maps equal to one at the crossing points of components of $\mathcal{Y}_v^{\text{red}}$.*

Proof of Theorem 1.1. For any fixed place v of $\mathbb{Q}$, and any component C_i of $\mathcal{Y}_v^{\text{red}}$, it follows from the definition that δ_{v,C_i} is the class that corresponds to the element $\delta = (\oplus 0) \oplus (\oplus a_x) \in (\oplus_{x \in C_i - Z} \mathbb{Z}) \oplus (\oplus_{x \in C_i \cap Z} K^*/U_x^1)$, where the a_x are as defined in Section 1. This term is independent of the choice of a divisor with identity residues at crossings, and it follows that the right-hand side of the equation in Theorem 2.1 is as well. Furthermore, it is clear that $\varepsilon(\mathcal{Y}_v, V)$ is independent of our choice of D'. It therefore follows from the theorem that $\varepsilon(D'_v, V)$ is independent of the choice of D'. Next, we note that $\varepsilon_{v,0}(D', V)$ must also be independent of our choice of D' (by [**7**, Lemma 3.3], for example). Therefore it must be the case that $\varepsilon_v(D', V) = \varepsilon_{v,0}(D', V)\varepsilon(D'_v, V)$ is independent of the choice of D'. The product of all of the $\varepsilon_v(D', V)$ is equal to $\varepsilon(D', V)$, which must be equal to 1, from the theorem of Fröhlich and Queyrut [**6**]. This tells us that we can define an element $\mu(\mathcal{X}, G, V)$ in $H^2(\mathbb{Q}, \mathbb{Z}/2\mathbb{Z})$ by setting the local component at the prime v to be equal to the sign of $\varepsilon_v(D', V)$. □

3. *The connection to $w_2(\pi)$*

Let $\pi : \mathcal{X} \longrightarrow \mathcal{Y}$ be a tamely ramified cover of degree n, where $\mathcal{X}$ and $\mathcal{Y}$ are regular schemes and $\mathcal{Y}$ is connected. Furthermore, we must make the technical assumption that the ramification indices are all odd. In [**1**], Cassou-Noguès, Erez and Taylor use Grothendieck's equivariant cohomology theory to define an invariant $w_2(\mathcal{X}/\mathcal{Y}) = w_2(\pi) \in H^2(\mathcal{Y}_{\text{et}}, \mathbb{Z}/2\mathbb{Z})$, associated to this situation. Their definition generalizes to define classes $w_i(\pi)$ that lie in $H^i(\mathcal{Y}_{\text{et}}, \mathbb{Z}/2\mathbb{Z})$ for all positive integers i, but in this paper we are interested only in w_2. These terms are generalized Stiefel–Whitney classes, and are obtained by pulling back the universal Hasse–Witt classes defined by Jardine in [**9**] and [**10**], using classifying maps related to a quadratic form E. The precise definition of E uses the existence of a locally free sheaf $\mathcal{D}_{\mathcal{X}/\mathcal{Y}}^{-1/2}$ whose square is the inverse different of the covering $\mathcal{X}/\mathcal{Y}$. In this section, we consider the relationship between the class $\mu(\mathcal{X}, G, V)$ lying in $H^2(\mathbb{Q}, \mathbb{Z}/2\mathbb{Z})$, which we defined in Theorem 1.1, and $w_2(\pi)$.

Let D' be a choice of a horizontal canonical divisor on $\mathcal{Y}$, in the sense of the previous sections, and let $i : D' \hookrightarrow \mathcal{Y}$ be the natural inclusion. An étale covering of $\mathcal{Y}$ naturally restricts to give an étale covering of D'. We now have the following natural maps:

$$i^* : H^2(\mathcal{Y}_{\text{et}}, \mathbb{Z}/2\mathbb{Z}) \longrightarrow H^2(D'_{\text{et}}, \mathbb{Z}/2\mathbb{Z}),$$
$$\text{res} : H^2(D'_{\text{et}}, \mathbb{Z}/2\mathbb{Z}) \longrightarrow H^2_{\text{et}}(\mathbb{Q}(D'), \mathbb{Z}/2\mathbb{Z}) = H^2_{\text{gal}}(\bar{\mathbb{Q}}/\mathbb{Q}(D'), \mathbb{Z}/2\mathbb{Z}),$$
$$\text{cor} : H^2_{\text{gal}}(\bar{\mathbb{Q}}/\mathbb{Q}(D'), \mathbb{Z}/2\mathbb{Z}) \longrightarrow H^2(\mathbb{Q}, \mathbb{Z}/2\mathbb{Z}),$$

where the latter two maps are a restriction and a corestriction in the sense of Serre (for details, see [**14**, Chapter VII]). Composing these maps gives a natural map

$$H^2(\mathcal{Y}_{\text{et}}, \mathbb{Z}/2\mathbb{Z}) \longrightarrow H^2(\mathbb{Q}, \mathbb{Z}/2\mathbb{Z}).$$

We denote the image of the class $w_2(\pi) \in H^2(\mathcal{Y}_{\text{et}}, \mathbb{Z}/2\mathbb{Z})$ under this map by $\tilde{w}_2(\pi) \in H^2(\mathbb{Q}, \mathbb{Z}/2\mathbb{Z})$. At first glance, it appears as though this element may depend on our choice of canonical divisor D'. However, we show that it does not depend on this choice, and furthermore that the element $\tilde{w}_2(\pi)$ is connected in a natural way to the element $\mu(\mathcal{X}, G, V)$. Recall that $\mu(\mathcal{X}, G, V)$ is defined by letting the local invariant at the place v be given by the sign of $\varepsilon(D'_v, V)$. This also initially appeared to depend on the choice of D', but turns out to be independent of the choice.

We observe that the class $\mu(\mathcal{X}, G, V)$ depends on the choice of a representation V of G, while $\tilde{w}_2(\pi)$ does not. The natural representation to consider is the regular representation of the group G, which we denote by R. In particular, the nicest possible theorem comparing the invariants would say that $\mu(\mathcal{X}, G, R) = \tilde{w}_2(\pi)$. However, we have shown only that $\mu(\mathcal{X}, G, V)$ is a well-defined class in the case where V is of dimension zero and of trivial determinant, neither of which holds for R. So, instead of setting $V = R$, we consider the representation $V = R - \det(R) - T^{n-1}$, where $\det(R)$, the determinant of the regular representation, is a character whose order is either one or two, T is the trivial representation, and n is the degree of the cover $\mathcal{X}/\mathcal{Y}$. This choice of V is an orthogonal representation, and it has trivial determinant and dimension 0. We can now prove the following theorem.

THEOREM 3.1. *Assume that we are in the above situation, and in particular that* $V = R - \det(R) - T^{n-1}$. *Let* $\mathcal{Y}_1/\mathcal{Y}$ *be either the trivial cover or the subcover of* $\mathcal{X}/\mathcal{Y}$ *of degree two, depending on whether* $\det(R)$ *is of order one or two respectively. Then, as classes in* $H^2(\mathbb{Q}, \mathbb{Z}/2\mathbb{Z})$, *we have the equality*

$$\mu(\mathcal{X}, G, V) = \tilde{w}_2(\mathcal{X}/\mathcal{Y}) - \tilde{w}_2(\mathcal{Y}_1/\mathcal{Y}) - (n-1)\tilde{w}_2(\mathcal{Y}/\mathcal{Y}).$$

Proof. The proof of this theorem relies on the interpretation of each side of the equation as a Stiefel–Whitney class. In particular, the construction of $w_2(\pi)$ in [**1**] first uses the classifying space of the orthogonal group to obtain an element in $H^2(\pi^{t,0}(\mathcal{Y}), \mathbb{Z}/2\mathbb{Z})$, which they then map to $H^2(\mathcal{Y}_{\text{et}}, \mathbb{Z}/2\mathbb{Z})$ using the canonical map. On the other hand, if we let K be the function field of the canonical divisor D', then there is a natural map from $\text{Gal}(\bar{K}/K)$ to $\pi^{t,0}(\mathcal{Y})$ that will induce a natural map from $H^2(\pi^{t,0}(\mathcal{Y}), \mathbb{Z}/2\mathbb{Z}) \longrightarrow H^2(\text{Gal}(\bar{K}/K), \mathbb{Z}/2\mathbb{Z})$. By the naturality of these maps, we see that the restriction of $i^*(w_2(\pi))$ is the Stiefel–Whitney class of the induced permutation orthogonal representation on $\text{Gal}(\bar{K}/K)$.

The fact that the corestriction of this Stiefel–Whitney class is $\tilde{w}_2(\pi)$ now follows from results of Deligne, which interpret local Stiefel–Whitney classes in terms of local root numbers. In particular, we define the local root number $W_v(V)$ to be the sign of $\varepsilon_v(\mathcal{Y}, V)$, and

$$\mathrm{cl} : H^2\left(\mathrm{Gal}\left(\frac{\overline{K(\mathcal{Y}_v)}}{K(\mathcal{Y}_v)}\right), \frac{\mathbb{Z}}{2\mathbb{Z}}\right) \longrightarrow (\mathbb{Q}/\mathbb{Z})_2$$

to be the class homomorphism, which will be an isomorphism in characteristic not equal to two. Then the following result is [**4**, Proposition 5.2].

LEMMA 3.2. *Let $d = 1$, so that the fibers $\mathcal{X}_v$ and $\mathcal{Y}_v$ are all one-dimensional schemes. Furthermore, let V be an orthogonal virtual representation of dimension zero and trivial determinant. Under these hypotheses, the local root number $W_v(V) = \exp(2\pi i\,\mathrm{cl}(sw_v))$, where sw_v is the local Stiefel–Whitney class, and $\mathrm{cl}(sw_v) \in \{0, 1/2\} \subset \mathbb{Q}/\mathbb{Z}$.*

In other words, in characteristic not equal to two, the sign of the ε-constants $\varepsilon_v(D', V)$ of the representation on the one-dimensional horizontal divisor D' are determined by whether or not the classes $w_2(\pi)$ are trivial in the Brauer group, and $\varepsilon_v(D', V)$ is automatically positive when $v = 2$. One can easily check that these are exactly the terms that are coming up in the computation of the class of Cassou-Noguès, Erez and Taylor.

In particular, $\varepsilon_v(D', V) = \varepsilon_v(D', R)\varepsilon_v(D', \det(R))$ is the same as the local Hasse–Witt invariants. However, we are working with étale covers of curves, and so from the results of [**1**] discussed above, these Hasse–Witt invariants are simply the images of the appropriate classes $w_2(\pi)$. This proves Theorem 3.1. □

4. *Proof of Lemma* 2.2

The proof of Lemma 2.2 involves a generalized version of Bertini's theorem. For now, let us assume that X is a smooth curve defined over an infinite field k, and let us choose a finite set of points $p_1, \dots, p_m$ on X. We define the divisor $p = \sum_i p_i$. Furthermore, let us choose constants c_i that lie in the residue field $k(p_i)$ of the points p_i. Finally, let us choose Λ to be an effective very ample divisor on X of large degree, which is supported off p. We look at the group of global sections $H^0(X, \mathcal{O}_X(\Lambda))$, and we let $f_0, \dots, f_t$ be a basis of this group. This basis gives us a projective embedding from X into $\mathbb{P}^t_k$, whose projective coordinates we write as $x_0, \dots, x_t$.

We wish to prove that there exist linear forms l_0 and l_1 in the variables x_i such that the following properties hold.

(1) If H_i is the hyperplane defined by $l_i = 0$ in $\mathbb{P}^t_k$, then $H_i \cap X$ is a finite set of closed points that is regular and disjoint from $\{p_1, \dots, p_m\}$. Furthermore, we wish to choose the l_i so that $H_1 \cap H_2 \cap X$ is empty.

(2) It follows from (1) that the function $l_1/l_0|_X$ is in $\mathcal{O}_{X,p_i}$ for each i. However, we further wish to specify that the image of l_1/l_0 in $k(p_i)$ is the given constant c_i.

The classical version of Bertini's theorem [**8**, Theorem II.8.18] tells us that there exist linear forms l_0 such that H_0 satisfies condition (1). We now fix one choice of such an l_0, and we attempt to construct an l_1 such that the pair satisfies properties (1) and (2). We begin by looking at the set V consisting of all linear forms such that

$\{l_0, l_1\}$ satisfy condition (2). In other words,

$$V = \left\{ l = a_0x_0 + \ldots + a_tx_t \colon \forall j, \frac{l}{l_0}|_X(p_j) = c_j \in k(p_j) \right\}.$$

This V will be an affine space over k. Furthermore, because we chose the divisor Λ to have high degree, it follows from a Riemann–Roch argument that V is of codimension m inside $H^0(X, \mathcal{O}_X(\Lambda))$.

For each point $x \in X$, we now define a set $V_x \subseteq V$ that consists of all linear forms $l \in V$ such that the hyperplane defined by $l = 0$ has contact order greater than 1 at x. In other words, V_x will consist of those linear forms that do not intersect X nicely at the point x. We can again use the Riemann–Roch theorem to show that for almost all choices of x, we find that the dimension of V_x is equal to dim $V - 2$.

Let $U = X - \{p_1, \ldots, p_m\}$ so that U is an affine curve, and define $T \subseteq U \times V$ to be the set of all pairs (x, l) such that $x \in U$ and $l \in V_x$. We have seen that the projection map $\pi : T \longrightarrow U$ is surjective, and for almost all $x \in U$ (in particular, for those points such that $k(x) = k$) we see that the fiber $\pi^{-1}(x)$ is an affine space whose dimension is equal to dim $V - 2$. In particular, this shows that T is irreducible, and that the dimension of T is equal to dim $V - 1$. This shows us that the natural projection map $\gamma : T \longrightarrow V$ must not be surjective.

Therefore, we can choose some element $l_1 \in V$ that is not in the image of γ. In particular, the hyperplane H defined by $\{l_1 = 0\}$ is such that $H \cap U$ is regular and, since $l_1 \in V$, we know that $l_1/l_0(p_i) = c_i \in k(p_i)$, and thus that l_1 and l_o satisfy conditions (1) and (2) above.

So far, we have made the argument only for the case where X is a smooth curve. However, as long as X is a reduced curve with smooth irreducible components that have normal crossings, then the same argument will hold as long as we include these crossing points in the set of $\{p_i\}$. Instead of using the normal Riemann–Roch theorem, we now use the Riemann–Roch theorem for singular curves, described in [**8**, p. 298].

In order to prove Lemma 2.2, we apply this generalized version of Bertini's theorem to each $X = \mathcal{Y}_v^{\mathrm{red}}$. Specifically, we choose the set of points $\{p_1, \ldots, p_m\}$ to include the crossing points of components of $\mathcal{Y}_v^{\mathrm{red}}$, as well as the points in $D' \cap \mathcal{Y}_v^{\mathrm{red}}$. The above argument then allows us to find a meromorphic function h where we can specify the values of the function $h = l_1/l_0$ at the crossing points of $\mathcal{Y}_v^{\mathrm{red}}$ so that the residues that come up when we consider $D'' = D' + \mathrm{div}(h)$ are all equal to 1, and D'' intersects $\mathcal{Y}_v^{\mathrm{red}}$ in the desired fashion. □

Acknowledgements. I would like to thank Ted Chinburg for his insightful comments and suggestions, and the referee for helping to streamline the proof of Theorem 3.1.

References

1. P. Cassou-Noguès, B. Erez and M. Taylor, 'Invariants of a quadratic form attached to a tame covering of schemes', Colloque International de Théorie des Nombres (Talence, 1999) *J. Théorie Nombres Bordeaux* 12 (2000) 597–660.

2. T. Chinburg, B. Erez, G. Pappas and M. Taylor, 'ε-constants and the Galois structure of de Rham cohomology', *Ann. of Math.* (2) 146 (1997) 411–473.

3. P. Deligne, 'Les constantes des équations fonctionnelles des fonctions L', *Modular functions of one variable, II* (Proc. Internat. Summer School, Univ. Antwerp, Antwerp, 1972), Lecture Notes in Math. 349 (Springer, Berlin, 1973) 501–597.

4. P. Deligne, 'Les constantes locales de l'équation fonctionnelle de la fonction L d'Artin d'une représentation orthogonale', *Invent. Math.* 35 (1976) 299–316.
5. A. Fröhlich, 'Orthogonal representations of Galois groups, Stiefel–Whitney classes and Hasse–Witt invariants', *J. Reine Angew. Math.* 360 (1985) 84–123.
6. A. Fröhlich and J. Queyrut, 'On the functional equation of the Artin L-function for characters of real representations', *Invent. Math.* 20 (1973) 125–138.
7. D. Glass, 'Epsilon constants and orthogonal representations', *Compositio Math.* 140 (2004) 1135–1148.
8. R. Hartshorne, *Algebraic geometry*, Grad. Texts in Math. 52 (Springer, New York/ Heidelberg, 1977).
9. J. Jardine, 'Universal Hasse–Witt classes', *Algebraic K-Theory and algebraic number theory*, Contemp. Math. 83. (Amer. Math. Soc., Providence, RI, 1989) 83–100.
10. J. Jardine, 'Cohomological invariants associated to symmetric bilinear forms', *Exposition. Math.* 10 (1992) 97–134.
11. S. Saito, 'Functional equations of L-functions of varieties over finite fields', *J. Fac. Sci. Univ. Tokyo Sect. IA Math.* 31 (1984) 287–296.
12. T. Saito, 'ε-factor of a tamely ramified sheaf on a variety', *Invent. Math.* 113 (1993) 389–417.
13. T. Saito, 'The sign of the functional equation of the L-function of an orthogonal motive', *Invent. Math.* 120 (1995) 119–142.
14. J.-P. Serre, *Local fields*, translated from the French by Marvin Jay Greenberg, Grad. Texts in Math. 67 (Springer, New York/Berlin, 1979).

Darren Glass
Department of Mathematics
Columbia University
New York, NY 10027
USA

glass@math.columbia.edu