Problems of the Month
Gettysburg College Mathematics Department
November, 2005

Details: Solutions should be submitted to the icosahedron in the math department by 5 pm on November 30th, at which point the math department faculty will review the solutions. You are encouraged to work together, and submissions from groups will be accepted. We would also love to see any work you do on any of the problems, even if you do not submit full solutions. If you have any questions, or would like to suggest problems of your own, please contact Darren Glass in Office 215E or at dglass@gettysburg.edu.

N1. The 2500 Gettysburg College students each knows a unique tidbit of gossip which is not known to any of the others. These gossips communicate only by sending email, and when someone sends an email, he or she will not only reveal the original piece of gossip they knew, but any other juicy details he or she has heard in the meantime. Find the minimum number of emails that will suffice to share every piece of gossip with all of the gossips.

N2. Let $f$ and $g$ be odd functions (that is, $f(-x) = -f(x)$ and $g(-x) = -g(x)$ for all $x$) that are infinitely differentiable at $x = 0$, and assume that $f'(0) = g'(0) = 1$. Consider the compositions $F(x) = f(g(x))$ and $G(x) = g(f(x))$; we’ll be comparing their derivatives at $x = 0$.

a) Show that $F'(0) = G'(0)$.

b) Show that for all even $n$, $F^{(n)}(0) = G^{(n)}(0)$.

c) Is it always true that for all odd $n$, $F^{(n)}(0) = G^{(n)}(0)$? If so, prove it; if not, give a counterexample.

N3. (from the 1999 HiMCM) Major thoroughfares in big cities are usually highly congested. Traffic lights are used to allow cars to cross the highway or to make turns onto side streets. During commuting hours, when the traffic is much heavier than on any cross street, it is desirable to keep traffic flowing as smoothly as possible. Consider a two-mile stretch of a major thoroughfare with cross streets every city block. Build a mathematical model that satisfies both the commuters on the thoroughfare as well as those on the cross streets trying to enter the thoroughfare as a function of the traffic lights. Assume there is a light at every intersection along your two-mile stretch. (Hint: You may want to start by assuming that the city blocks are of constant length and then generalize to blocks of variable length.)