Problems of the Month
Gettysburg College Mathematics Department

**Details:** Solutions should be submitted to the icosahedron in the math department by 5 pm on January 31st, at which point the math department faculty will review the solutions. You are encouraged to work together, and submissions from groups will be accepted. We would also love to see any work you do on any of the problems, even if you do not submit full solutions. If you have any questions, or would like to suggest problems of your own, please contact Darren Glass in Office 215E or at dglass@gettysburg.edu.

H1. Find all square numbers which are 1 more than a prime number.

H2. Can you find two integers \( x \) and \( y \) so that \( x \times y = 2005 \) and \( x + y = 2006 \) (or prove that no such pairs exist)? Conversely, can you find two integers \( x \) and \( y \) so that \( x \times y = 2006 \) and \( x + y = 2005 \) (or prove that no such pairs exist)?

H3. Jack and Jill decide to play a game in which they take turns flipping a coin until one of them gets a head and the person who gets the head will be declared the winner. Given that Jack flips first, what is the probability that he wins the game?

H4. A can is in the shape of a (right circular) cylinder of radius \( r \) and height \( h \). In particular, the surface of the can consists of two congruent circular disks of radius \( r \) and the side piece where the label would be. An ant is at a point on the top circle (that forms the edge of the top of the can) and wants to crawl to the point on the bottom circle that is diametrically opposite to its starting point. As a function of \( r \) and \( h \), how far must the ant crawl? (You may not be able to prove rigorously that your answer is the best possible, but try to give as much justification as you can.)