Can you find any triplets $(p, p+2, p+4)$ such that all three numbers are prime? Do you think there are infinitely many? If yes, why? If no, why not?

The only prime triplet is $(3, 5, 7)$. In order to see this, we first note that the only prime p which is congruent to $0 \pmod 3$ is 3 itself. We can now proceed by considering several cases.

If $p \equiv 0 \pmod 3$ then as we noted above the only way p can be prime is if $p = 3$. In this case one can easily verify that $p + 2 = 5$ and $p + 4 = 7$ are also prime.

If $p \equiv 1 \pmod 3$ then $p + 2 \equiv 0 \pmod 3$ and therefore in order to be prime $p + 2$ would have to be equal to 3. However, this would imply that $p = 1$ and 1 is not a prime number. Therefore, there are no prime triplets with $p \equiv 1 \pmod 3$.

If $p \equiv 2 \pmod 3$ then $p + 4 \equiv 6 \equiv 0 \pmod 3$. Thus by above in order for $p + 4$ to be prime it would have to equal 3, which is impossible.

Because we know that p must be congruent to $0, 1,$ or $2 \pmod 3$ we have now exhausted all possibilities and shown that the only prime triplet is $(3, 5, 7)$.