

2. We should use the Euclidean Algorithm to do this problem. In particular, this will tell us that  $24 * 89 - 35 * 61 = 1$ . In other words,  $-35 * 61 \equiv 1 \pmod{89}$ , so the inverse is  $-35 \equiv 54$ .

3. If  $x^2 \equiv 1 \pmod{p}$ , then  $p|(x^2 - 1)$ . This implies (why?) that  $p|(x - 1)$  or  $p|(x + 1)$ . In other words, wither  $x \equiv 1$  or  $x \equiv -1 \pmod{p}$ .

5. Fermat's Little Theorem tells us that  $a^{p-1} \equiv 1 \pmod{p}$ . But  $a^{p-1} = (a^k)^2$ , so this tells us that  $(a^k)^2 \equiv 1$ . The result follows from Problem 3 above.

6. There are plenty of counterexamples.

9. Let  $d = (a, b)$ . By definition,  $d|b$  and therefore  $d$  is a common divisor of  $(a, b)$  and  $b$ . But  $d$  is the greatest divisor of itself, and therefor it must be the GREATEST common divisor of  $(a, b)$  and  $b$ .

10. Lots of counterexamples