

1. Suppose $(a, b) = 1$, $a|c$, and $b|c$. Show that $ab|c$.

Because $(a, b) = 1$ we know that there exist x and y such that $ax + by = 1$. Thus, $acx + bcy = c$. Now, $b|c$ so it follows that $ab|ac$ and $a|c$ so it follows that $ab|bc$, but this implies that ab divides all linear combinations of ac and bc so in particular it must divide $acx + bcy = c$.

Alternatively, from the hypotheses we know that there exists some integer m such that $c = ma$. Given that $b|ma$ and $(b, a) = 1$ it follows from class that $b|m$. In particular there exists some integer n so that $m = bn$. But now $c = am = abn$ and thus $ab|c$.

5. Prove that if $(a, m) = (b, m) = 1$ then $(ab, m) = 1$

Assume not. Then there must exist some prime p such that $p|(ab, m)$. In particular, this implies that $p|m$ and $p|ab$. Because p is prime the latter fact implies that either $p|a$ or $p|b$ (or both). But this in turn implies that either $p|(a, m)$ or $p|(b, m)$ contradicting our hypothesis.

6. The correct answer to part d was that $\nu(p_1^{e_1} \dots p_r^{e_r}) = (e_1 + 1)(e_2 + 1) \dots (e_r + 1)$. The answers to parts a-c are just special cases of this formula. This formula can be proven in several different ways.