

Calculus I
Professor Glass
Practice Midterm #1

These problems are meant to give you a sense of the kinds of questions I think are important and that I think you should be able to answer. I make no claims that the problems on the actual exam will resemble these problems, and I certainly do not expect you to be able to do all of these problems in less than two hours.

1. Find the domain, range, and inverse of each of the following functions.

$$a) f(x) = \sqrt{e^x + 2}$$

$$b) g(x) = \frac{x - 2}{x + 2}$$

$$c) h(x) = 7 + 2^{x^2 - 5}$$

2. Suppose that the number of bacteria in a culture at time t is given by $x = f(t) = 10^2 e^{2t+1}$. What is $f^{-1}(t)$? What does this function represent in words?

3. Without using your calculator, sketch each of the following graphs based on the graphs of simple functions. Show all intermediate steps.

$$a) y = 2e^{x+1} + 5$$

$$b) y = 3 \sin(2x)$$

$$c) y = |\ln(x + 5)|$$

$$d) y = 2 - (x + 5)^2$$

4. Calculate the following limits or explain why they do not exist.

$$a) \lim_{x \rightarrow 3} \frac{|x - 3|}{2x - 6}$$

$$b) \lim_{x \rightarrow -1} \frac{x^4 - 1}{x^2 - 1}$$

$$c) \lim_{h \rightarrow 3} \frac{1/h - 1/3}{h - 3}$$

$$d) \lim_{x \rightarrow 1} \frac{x - \sqrt{2x^2 - 1}}{x - 1}$$

5. A scientist dumped 500 animals of a certain species on a deserted island. Let $P(t)$ be the population of the colony t months later. For this species of animal it is known that the population doubles every 14 months and grows exponentially. Find an exact formula for $P(t)$ and determine how long it takes for the population to reach 3000 animals.

6. Use the Intermediate Value Theorem to show that $f(x) = x^3 - 5x - 7$ has a root between 2 and 3.

7. In class (and in the book) we discussed how there are three parts to the definition of continuity, and thus there are three different ways that a function can fail to be continuous at a point x . Give examples of each of these.

8. Let $f(x) = \frac{x^2 + 3x - 4}{x^2 + 7x + 12}$. Find $\lim_{x \rightarrow a} f(x)$ if $a = 2, -3, -4$, and ∞ .

9. At what values of x is the following function continuous:

$$f(x) = \begin{cases} x + 4, & x \leq -1 \\ x^2, & -1 < x < 1 \\ 2 - x, & x \geq 1 \end{cases}$$

10. Find two functions, $f(t)$ and $g(t)$ so that $f(g(t)) = 1 - 2\cos^2(x)$.