How to Become President with less than 25% of the Popular Vote

A Word Problem from the U.S. Constitution

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March 19, 2009

You never know what will happen when you give a talk.

Article II, Section 1, Clause 2 of the U.S. Constitution:

Each State shall appoint, in such Manner as the Legislature thereof may direct, a Number of Electors, equal to the whole Number of Senators and Representatives to which the State may be entitled in the Congress: but no Senator or Representative, or Person holding an Office of Trust or Profit under the United States, shall be appointed an Elector.

Two Approaches:

- Theoretical - We'll use a 1961 paper by George Pòlya as a starting point.
- Empirical - We have data. We have a computer. Let's play around with this problem.

Theoretical Approach

Updated argument based on Pòlya's article *The minimum fraction of the popular vote that can elect the President of the United States* from the March, 1961 edition of *The Mathematics Teacher*.

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2. Each state awards all its electoral votes to the winning candidate in that state.
3. The number of votes cast in a state is *exactly* proportional to the number of U.S. representatives from that state.

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- N = the proportionality constant assumed in Simplifying Assumption 3 ($T = Nr$).
- W = the number of votes cast for the winning candidate.

Goal:

Minimize $\frac{W}{T}$.

Some Algebra (the high school kind)

$$T = 436N \tag{1}$$

If the winning candidate wins s states, the following inequality must be true:

$$(r_1 + 2) + (r_2 + 2) + (r_3 + 2) + \cdots + (r_s + 2) \geq 270$$

which can be rewritten as

$$r_1 + r_2 + r_3 + \cdots + r_s \geq 270 - 2s \tag{2}$$

The number of votes the winning candidate receives from winning these states is:

$$W \geq \left(\frac{r_1 N}{2} + 1\right) + \left(\frac{r_2 N}{2} + 1\right) + \cdots + \left(\frac{r_s N}{2} + 1\right)$$

This simplifies to

$$W \geq \frac{N}{2}(r_1 + r_2 + \cdots + r_s) + s \tag{3}$$

Dividing (3) by (1),

$$\frac{W}{T} \geq \frac{\frac{N}{2}(r_1 + r_2 + \cdots + r_s)}{436N} + \frac{s}{436N}$$

$$\frac{W}{T} \geq \frac{(r_1 + r_2 + \cdots + r_s)}{872} + \frac{s}{436N}$$

And using (2),

$$\frac{W}{T} \geq \frac{270 - 2s}{872} + \frac{s}{436N}$$

How would you minimize this?

Electoral Votes	Number of States	Number of Electoral Votes	Running Total of States	Running Total of Electoral Votes
3	8	24	8	24
4	5	20	13	44
5	5	25	18	69
6	3	18	21	87
7	4	28	25	115
8	2	16	27	131
9	3	27	30	158
10	4	40	34	198
11	4	44	38	242

Winning the 38 smallest states will earn a candidate 242 electoral votes. To get the 28 more needed for election, win

- Virginia (13 electoral votes)
- Any one of Georgia, New Jersey, or North Carolina (15 electoral votes)

Therefore $s = 40$.

The theoretical minimum percentage is then

$$\frac{W}{T} \geq \frac{270 - 2(40)}{872} + \frac{40}{436N}$$

Anyone want to worry about the second term?

$$\frac{270 - 2(40)}{872} = \frac{190}{872} \approx 0.2178899$$

Conclusion: A president can be elected with about 21.8% of the vote.

Halftime Entertainment

- How can you tell if a mathematician is an extrovert?

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- How can you tell if a mathematician is an extrovert?
- He looks at **your** shoes when he talks to you.

Empirical Approach

Really?

Pòlya's paper was written long before the general availability of spreadsheets. We don't have that excuse, so let's look at the data for the 2008 election and see if we can create a scenario that verifies Pòlya's conclusion.

Ideas for Further Research

1. Verify that 21.83% is the smallest percentage for the 2008 election. This is a non-trivial programming exercise.
2. See if the answer changes if we consider the electoral vote distribution method used by Maine and Nebraska.