Minimum Upsets Method

Introduction

The minimum upsets method's purpose is to come up with a ranking of teams that minimizes the number of upsets, as its title suggests. But what is an upset? It is when a team is ranked above a team that it lost to at some point during the season. The purpose of this method is to find a ranking where as few teams as possible are ranked above teams that beat them. So if you have a league with teams A, B and C, and A beats B, B beats C and A beats C, then the ranking should clearly be:

1. A
2. B
3. C

But if in the same league A beats B, B beats C, and then C beats A we have what is called a triad. This means there is no way to rank the reams with no upsets, because no matter how you rank these three teams one will be ranked higher than a team they lost to. This is one of the tricky things to deal with when working with this ranking method.

An Example

Now lets look at a more specific example of five teams from the 2012 Division III college football season: Johns Hopkins, Franklin & Marshall, Gettysburg, Dickinson and McDaniel. Consider following chart of these teams' (in the rows) wins and losses against teams (in the collumns):

Teams	Johns Hopkins	F & M	Gettysburg	Dickinson	McDaniel
Johns Hopkins	-	Loss, 12 - 14	Win 49-35	Win 49-0	Win 49-7
F & M	Win, 14 - 12	-	Loss, 31-38	Win 36-28	Win 35-10
Gettsyburg	Loss 35-49	Win, 38-31	-	Loss 23-13	Win 35-3
Dickinson	Loss 0-49	Loss 28-36	Win 13-23	-	Win 38-31
McDaniel	Loss 7-49	Loss 10-35	Loss 3-35	Loss 31-38	-

There are several ways to rank these teams, not looking to minimize upsets. Look at the following one:

1. Johns Hopkins
2. Franklin and Marshall
3. Gettysburg
4. Dickinson
5. McDaniel

This ranking has 3 upsets because Franklin and Marshall beat Johns Hopkins but is ranked below them, Gettysburg beat Franklin and Marshall but is ranked lower, and Dickinson beat Gettysburg and is ranked lower. In order to minimize upsets, lets try to rearrange the above ranking.
Try this ranking instead:

1. Franklin and Marshall
2. Johns Hopkins
3. Dickinson
4. Gettysburg
5. McDaniel

This ranking has only one upset since Gettysburg beat Franklin and Marshall but is ranked lower. We know that one upset is the best that we can do because this five team example has a triad: Gettsburg beat Franklin and Marshall, Franklin and Marshall beat Johns Hopkins and Johns Hopkins beat Gettysburg. Therefore there will be no way to rank these teams with no upsets, so this is the best we can do when minimizing upsets.
It is important to note that a minimum upsets ranking is not necessarily unique. In this situation it is unique, but there are several very different ways to get 2 upsets in a ranking. For example both these rankings have two upsets:

1. Johns Hopkins
2. Gettysburg
3. Franklin and Marshall
4. Dickinson
5. McDaniel

This has the upsets where Franklin and Marshall beat Johns Hopkins, and Dickinson beat

Gettysburg.

1. Franklin and Marshall
2. Johns Hopkins
3. Dickinson
4. McDaniel
5. Gettysburg

This has the upsets where Gettysburg College beat both Franklin and Marshall and McDaniel.
It is pretty easy to dismiss this second ranking as nonsense as McDaniel, who won no games is not ranked last, but it does still only have 2 upsets. This is one of the issues with this method, it is often considered too subjective.

The Math

When it comes to applying this method to a much larger example, say all of Division III football, it becomes much more complex. It would be almost impossible to have a ranking with no upsets, as triads are quite common in most sports. So the goal of this method is to get as few upsets as possible. To do this, we need to employ some programming skills. The trick with this method is to use some randomness to our advantage.
The first step when attempting to find a minimum upsets ranking is to create a matrix, A, that has all teams listed on the rows and columns. The a_{ij} entry, which is the entry in the ith row and the jth column, will be a 1 if team i beat team j, and it will be a 0 if team i lost to team j or if the two teams didn't play. Then a good place to start is to place the teams in a basic win loss order, placing the teams that won the most games at the top of the matrix, and those that lost the most at the bottom. This involves switching the rows and columns associated with a given two teams in the matrix. After this is complete, we begin to simply randomly swap rows and their associated columns, each time checking if the number of upsets has gone down or not from the previous time. If it has gone down, we keep the swap that just happened, if not we switch the two rows and columns back and try again. We can also have a swap kept if the number of upsets is equal to or less than the previous number, and only undo a swap if the number of upsets has increased.
Lets try this method on our small five team example that we discussed earlier. Lets begin by defining our A matrix, that has the teams listed in alphabetical order from top to bottom and right to left.

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

After switching the rows and columns to the order that gave us only one upset before, which is:

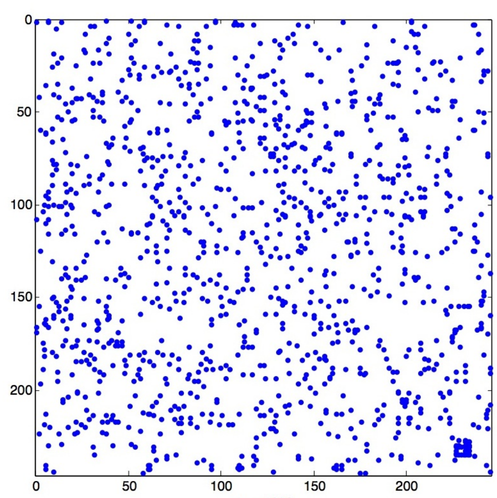
1. Franklin and Marshall
2. Johns Hopkins
3. Dickinson
4. Gettysburg
5. McDaniel.

Our new A matrix after these swaps occur is the following:

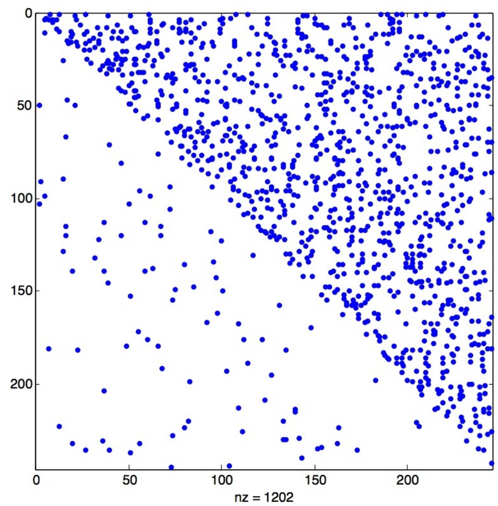
$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

It is clear that most of the 1's in the matrix appear in the top right hand corner. This is what we are looking for from a visual perspective. This configuration will minimize upsets because it means that the teams that have won, will be ranked above the teams that have lost, no matter what example or matrix we are working with.

Let's look at this on a much larger scale, namely all of the Division III football teams from the 2012 season. Here we have the Matlab spy command of the original win-loss matrix where the teams are listed in alphabetical order. This command shows a dot wherever there is a 1 in the matrix, and leaves the rest blank.



After we apply our program to sort these teams to minimize these violations the spy command gives us the following.



Clearly, the ones have mostly shifted to be in the upper right hand triangle of the matrix, as desired.
When looking for the minimum number of upsets, after trying many of these team swaps, we will count the number of upsets, and try to get that number as low as possible. This number will obvioulsy vary depending on the sport we are analyzing, the particular season and the luck involved in the random swaps. Therefore we do not have a way to find the best possible rankings that actually give us the lowest possible number of upsets, but we do have a way to get close to that number and improve our minimum upsets ranking as such.

Advantages and Disadvantages

The advantages of this method, compared to some others is that it is much more concrete than many other ranking methods. It is clear how the method works, and on small examples it is easy to compute by hand. This makes it more applicable and understandable. It also gives a very clear goal of its ranking.
One disadvantage of this ranking is that the actual number of minimum upsets is very difficult to calculate. Another, is that even if this number is attained, there may be many rankings that satisfy number, meaning there is some level of bias that can enter this ranking.

Possible Adjustments

There are some small adjustments we could make to this method in order to make slight improvements, or in order to have it suit our purposes a bit better. These improvements involve changing the definition of a upset.
The first improvement involves minimizing the sum of the upset rank differences. This means that a upset would count more if 20 ranked team beat a 5 ranked team, instead of if a 5 ranked team beat a 4 ranked team. What this ranking would do is attempt to minimize the number of “large upsets” and would care much less about smaller upsets. This would allow

more freedom in the ranking method.
The other possible improvement would be to only consider when a team beats a team ranked n above them a upset. What this does is allow the number of upsets in the final ranking to be even smaller. The number n can be determined based on the sport, or the particular set of teams being considered. The advantages of this method is that it would allow for small upsets to count at all, which would be like discounting things like rivalries, home field advantage, and other small factors. But this ranking would still include "important" upsets between teams with a large ranking gap.

Conclusion

Overall this method allows for a much more concrete interpretation of game data, and has a very clear goal in mind. Its potential for bias and difficulty to compute is frustrating to some, but its application is used in many sports, and works effectively to rank teams in a more comprehensible way.

Additional Reading

If you would like to learn more about the specifics of this method, here are some links to papers and websites that explore the topic more thoroughly than we do here.

This first link is to an article that explains more of the math behind this method.
A Minimum Violations Ranking Method
This second link is to Jay Coleman's actual minimum upsets ranking for the current Division I college football season.
MinV College Football Ranking