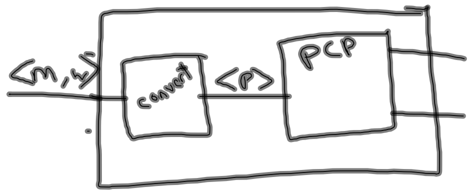
Post Correspondence Problem (PCP)

$$\left\{ \left[\frac{b}{ca}\right], \left[\frac{a}{ab}\right], \left[\frac{ca}{a}\right], \left[\frac{abc}{c}\right] \right\}$$

$$\left[\frac{a}{ab}\right]\left[\frac{b}{ca}\right]\left[\frac{ca}{a}\right]\left[\frac{a}{ab}\right]\left[\frac{abc}{c}\right]$$

PCP is undecidable

Proof: reduction from A_{TM}



P has a match iff M accepts w.

work w/ modified PCP (MPCP)

construct P' (to simulate M on w)

For a machine M $(Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$

on input $w = w_1 w_2 \dots w_n$

1. Starting domino

$$\left[\frac{\#}{\# q_0 w_1 \dots w_n \#}\right]$$

2. Transitions that move Right

$\forall a, b \in \Gamma$ and $q, r \in Q$ where $\cancel{q}\, r \neq q_{rej}$

if $\delta(q, a) = (r, b, R)$

add $\left[\frac{q\,a}{b\,r}\right]$ to P'

3. Transitions to the Left

$\forall a, b, c \in \Gamma$, $q, r \in Q$, $\cancel{q}\, r \neq q_{rej}$

if $\delta(q, a) = (r, b, L)$

add $\left[\frac{c\,q\,a}{r\,c\,b}\right]$ to P'

4. Other parts of the tape

$\forall a \in \Gamma$

add $\left[\frac{a}{a}\right]$ to P'

5. add $\left[\dfrac{\#}{\#}\right]$ and $\left[\dfrac{\#}{\sqcup\#}\right]$ to P'

$q\,a\#$

$b\,r\,\sqcup\#$

6. $\forall a \in \Gamma$

$$\left[\dfrac{a\,q_{acc}}{q_{acc}}\right] \text{ and } \left[\dfrac{q_{acc}\,a}{q_{acc}}\right]$$

7. $\left[\dfrac{q_{acc}\,\#\#}{q_{acc}\,\#}\right]$

Convert MPCP, P' to PCP, P

force a match P to start w/ P' start domino

$$\star u = * u_1 * u_2 * \dots * u_n$$

$$u \star = u_1 * u_2 * \dots * u_n *$$

$$\star u \star = * u_1 * u_2 * \dots * u_n *$$

For the start tile in P' $\left[\dfrac{t_s}{b_s}\right]$

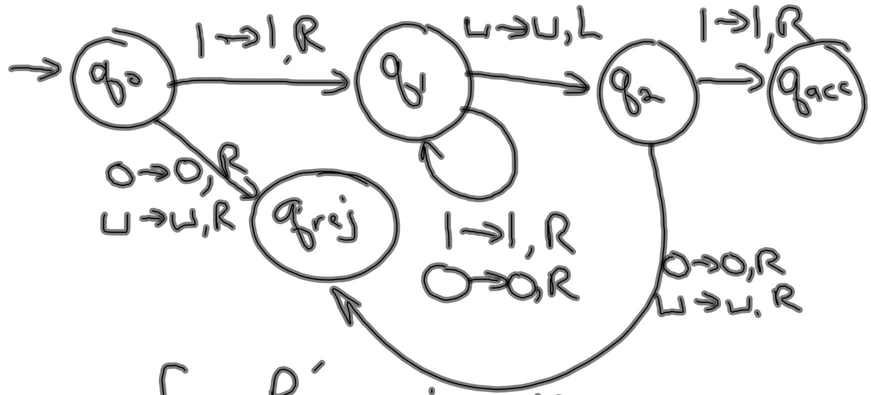
add $\left[\dfrac{\star t_s}{\star b_s \star}\right]$ to P

for all other tiles in P' $\left[\dfrac{t}{b}\right]$

add $\left[\dfrac{\star t}{b \star}\right]$ to P

add $\left[\dfrac{\star \diamond}{\diamond}\right]$ to P

M



Dominos for P' - sim. M on $w = 101$

1. $\left[\dfrac{\#}{\# q_0 w_1 \ldots w_n \#}\right] \Rightarrow \left[\dfrac{\#}{\# q_0 101 \#}\right]$

2. Right

$\left[\dfrac{q_0 1}{1 q_1}\right]$ $\left[\dfrac{q_1 1}{1 q_1}\right]$ $\left[\dfrac{q_1 0}{0 q_1}\right]$ $\left[\dfrac{q_2 1}{1 q_{acc}}\right]$

3. Left

$\left[\dfrac{0 q_1 \sqcup}{q_2 0 \sqcup}\right]$ $\left[\dfrac{1 q_1 \sqcup}{q_2 1 \sqcup}\right]$ $\left[\dfrac{\sqcup q_1 \sqcup}{q_2 \sqcup \sqcup}\right]$

4. $\left[\dfrac{0}{0}\right]$ $\left[\dfrac{1}{1}\right]$ $\left[\dfrac{\sqcup}{\sqcup}\right]$

5. $\left[\dfrac{\#}{\#}\right]$ $\left[\dfrac{\#}{\sqcup \#}\right]$

6. $\left[\dfrac{0 \; q_{acc}}{q_{acc}}\right]$ $\left[\dfrac{1 \; q_{acc}}{q_{acc}}\right]$ $\left[\dfrac{\sqcup \; q_{acc}}{q_{acc}}\right]$ $\left[\dfrac{q_{acc} \; 0}{q_{acc}}\right.$

. . . .

7 $\left[\dfrac{q_{acc} \# \#}{q_{acc} \#}\right]$