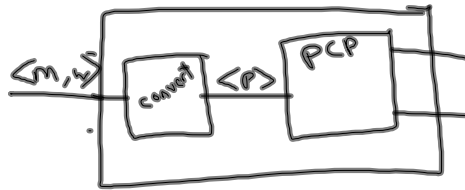


# Post Correspondence Problem (PCP)

$$\left\{ \left[ \frac{b}{ca} \right], \left[ \frac{a}{ab} \right], \left[ \frac{ca}{a} \right], \left[ \frac{abc}{c} \right] \right\}$$

$$\left[ \frac{a}{ab} \right] \left[ \frac{b}{ca} \right] \left[ \frac{ca}{a} \right] \left[ \frac{a}{ab} \right] \left[ \frac{abc}{c} \right]$$

PCP is undecidable  
 Proof: reduction from  $A_{im}$



$P$  has a match iff  
 $M$  accepts  $w$ .

work w/ modified PCP (MPCP)  
 construct  $P'$  (to simulate  $M$  on  $w$ )

For a machine  $M (Q, \Sigma, \Gamma, \delta, q_0, q_{acc}, q_{rej})$

on input  $w = w_1 w_2 \dots w_n$

1. Starting domino

$$\left[ \begin{array}{c} \# \\ \hline \# q_0 w_1 \dots w_n \# \end{array} \right]$$

2. Transitions that move Right

$$\forall a, b \in \Gamma \text{ and } q, r \in Q \text{ where } q \neq q_{rej}$$

$$\text{if } \delta(q, a) = (r, b, R)$$

$$\text{add } \left[ \begin{array}{c} qa \\ \hline br \end{array} \right] \text{ to } P'$$

3. Transitions to the Left

$$\forall a, b, c \in \Gamma \text{ and } q, r \in Q \text{ where } q \neq q_{rej}$$

$$\text{if } \delta(q, a) = (r, b, L)$$

$$\text{add } \left[ \begin{array}{c} cqa \\ \hline rcb \end{array} \right] \text{ to } P'$$

4. Other parts of the tape

$$\forall a \in \Gamma \text{ add } \left[ \begin{array}{c} a \\ \hline a \end{array} \right] \text{ to } P'$$

5. add  $\left[ \frac{\#}{\#} \right]$  and  $\left[ \frac{\#}{\perp \#} \right]$  to  $\mathcal{P}$ .

$g a \#$   
 $b r \perp \#$

6.  $\forall a \in \Gamma$

$\left[ \frac{a g_{acc}}{g_{acc}} \right]$  and  $\left[ \frac{g_{acc} a}{g_{acc}} \right]$

7.  $\left[ \frac{g_{acc} \#\#}{g_{acc} \#} \right]$

Convert MPCP,  $P'$  to PCP,  $P$   
 force  $\overset{\text{a match}}{n}$   $P$  to start w/  $P'$  start  
 domino

$$\begin{aligned} \star u &= \star u_1 \star u_2 \star \dots \star u_n \\ u \star &= u_1 \star u_2 \star \dots \star u_n \star \\ \star u \star &= \star u_1 \star u_2 \star \dots \star u_n \star \end{aligned}$$

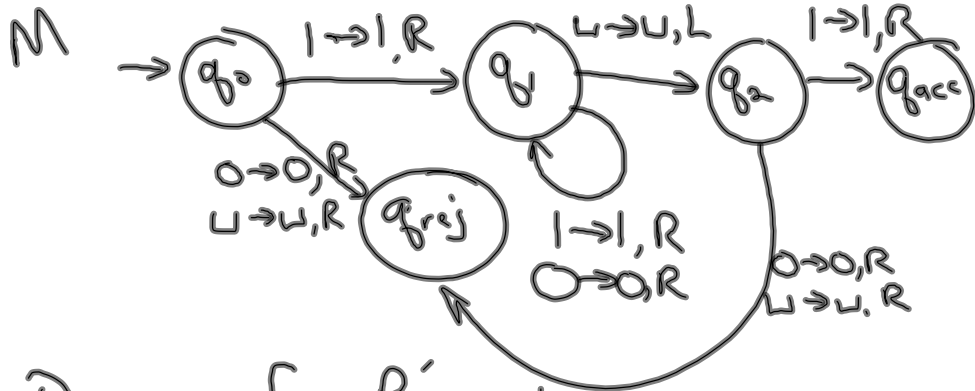
for the start tile in  $P'$   $\begin{bmatrix} t_s \\ b_s \end{bmatrix}$

add  $\begin{bmatrix} \star t_s \\ \star b_s \star \end{bmatrix}$  to  $P$

for all other tiles in  $P'$   $\begin{bmatrix} t \\ b \end{bmatrix}$

add  $\begin{bmatrix} \star t \\ b \star \end{bmatrix}$  to  $P$

add  $\begin{bmatrix} \star \diamond \\ \diamond \end{bmatrix}$  to  $P$



Dominos for  $P'$  - sim.  $M$  on  $w = 101$

$$1. \left[ \begin{array}{c} \# \\ \hline \#q_0w_1 \dots w_n\# \end{array} \right] \Rightarrow \left[ \begin{array}{c} \# \\ \hline \#q_0101\# \end{array} \right]$$

$$2. \text{Right} \quad \left[ \begin{array}{c} q_01 \\ \hline 1q_1 \end{array} \right] \left[ \begin{array}{c} q_11 \\ \hline 0q_1 \end{array} \right] \left[ \begin{array}{c} q_10 \\ \hline 0q_1 \end{array} \right] \left[ \begin{array}{c} q_21 \\ \hline 1q_{acc} \end{array} \right]$$

$$3. \text{Left} \quad \left[ \begin{array}{c} 0q_11 \\ \hline q_201 \end{array} \right] \left[ \begin{array}{c} 1q_11 \\ \hline q_211 \end{array} \right] \left[ \begin{array}{c} 1q_11 \\ \hline q_211 \end{array} \right]$$

$$4. \left[ \begin{array}{c} 0 \\ \hline 0 \end{array} \right] \left[ \begin{array}{c} 1 \\ \hline 1 \end{array} \right] \left[ \begin{array}{c} 1 \\ \hline 1 \end{array} \right]$$

$$5. \left[ \begin{array}{c} \# \\ \hline \# \end{array} \right] \left[ \begin{array}{c} \# \\ \hline 1\# \end{array} \right]$$

$$6. \left[ \begin{array}{c} 0q_{acc} \\ \hline q_{acc} \end{array} \right] \left[ \begin{array}{c} 1q_{acc} \\ \hline q_{acc} \end{array} \right] \left[ \begin{array}{c} 1q_{acc} \\ \hline q_{acc} \end{array} \right] \left[ \begin{array}{c} q_{acc}c \\ \hline q_{acc} \end{array} \right] \dots$$

$$7. \left[ \begin{array}{c} q_{acc}\#\# \\ \hline q_{acc}\# \end{array} \right]$$