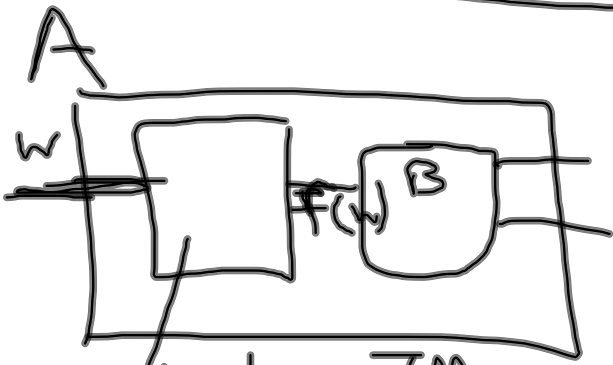
Polynomial Time Reducibility



poly time TM
starts on input w
halts with f(w)
on the tape

$A \leq_p B$

If $A \leq_p B$ and $B \in P$
Then $A \in P$.

3-SAT

CNF - conjunctive normal form

$(x \vee \bar{y} \vee z) \wedge (\ldots) \wedge (\quad)$

any Boolean formula can be expressed in CNF

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3 CNF

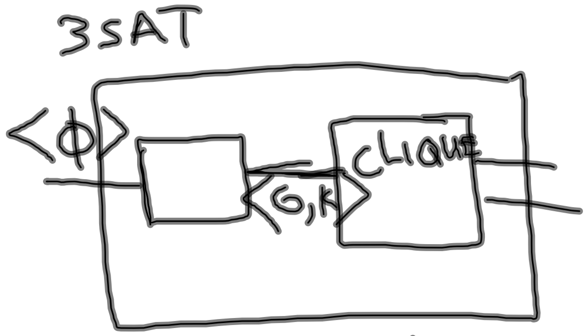
↳ each term has exactly 3 literals (e.g. $x, \bar{x}$)

term $(x_1 \vee \bar{x}_2 \vee \bar{x}_3)$

3SAT = $\{ \phi \mid \phi$ is a satisfiable 3CNF formula $\}$

CLIQUE = $\{ \langle G, k \rangle \mid$ graph G contains a k-clique $\}$

$3SAT \leq_p CLIQUE$



$$\phi = (a_1 \vee b_1 \vee c_1) \wedge (a_2 \vee b_2 \vee c_2) \wedge \ldots \wedge (a_k \vee b_k \vee c_k)$$

Construct G

k groups of 3 nodes

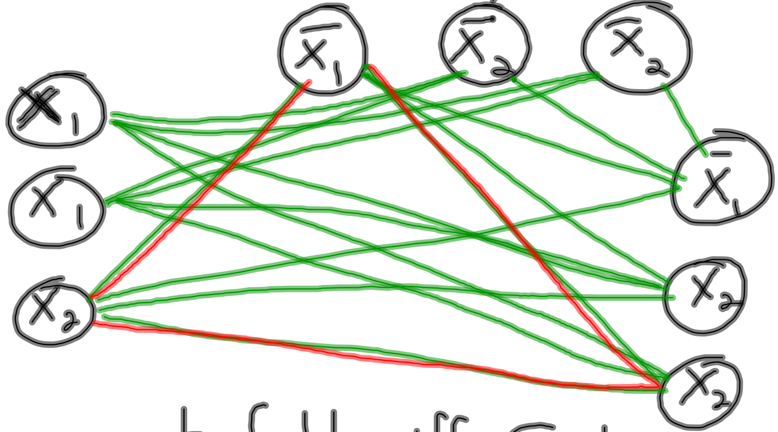
one node for each appearance of a literal in ϕ

Edges between every pair except

1. nodes in the same group
2. nodes w/ contradictory labels (literals) eg $x, \bar{x}$

$$\Phi = (x_1 \vee x_1 \vee x_2) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_2) \wedge (\bar{x}_1 \vee x_2 \vee x_2)$$

$x_2 = t$
$\bar{x}_1 = t$
$x_1 = f$



Φ is satisfiable iff G has a k-clique (k = # of terms in Φ)

I. Suppose Φ is satisfiable.
So each term will have at least one true literal.

Consider the nodes w/ the labels from those true literals
These form a clique in G

II. Suppose G has a k-clique
- no two of the clique's nodes are in the same group.
- each group contains exactly one of the clique's nodes
- assign values to variables s.t. the literals are true
- that assignment satisfies Φ.

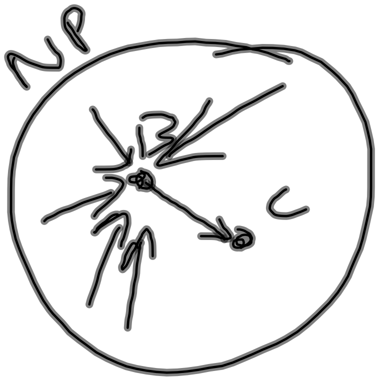
A language is NPC if two conditions are true:

1. The language is in NP.
2. Every language in NP is poly. time reducible to it.

NP-Hard

If B is NPC and $B \in P$, then $P = NP$.

If B is NPC and $B \leq_p C$ for C in NP, then C is NPC.



Cook-Levin Thm:

SAT is NP-Complete.