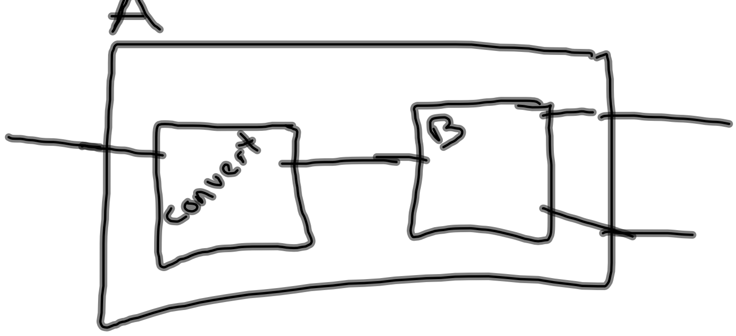
Reducibility

problem A reduces to problem B



if A is reducible to problem B and B is decidable, then A is decidable

if A is reducible to B and A is not decidable, then B must be undecidable.

$HALT_{TM} = \{\langle M, w\rangle \mid M$ is a TM and M halts on input $w\}$

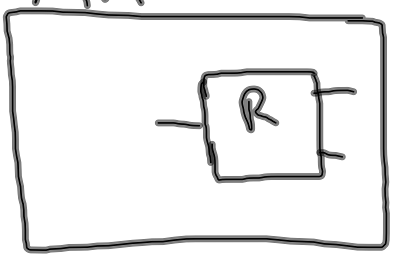
$HALT_{TM}$ is undecidable:

proof (by contradiction)

Suppose $HALT_{TM}$ is decidable and TM R decides it.

Construct S to decide A_{TM}.

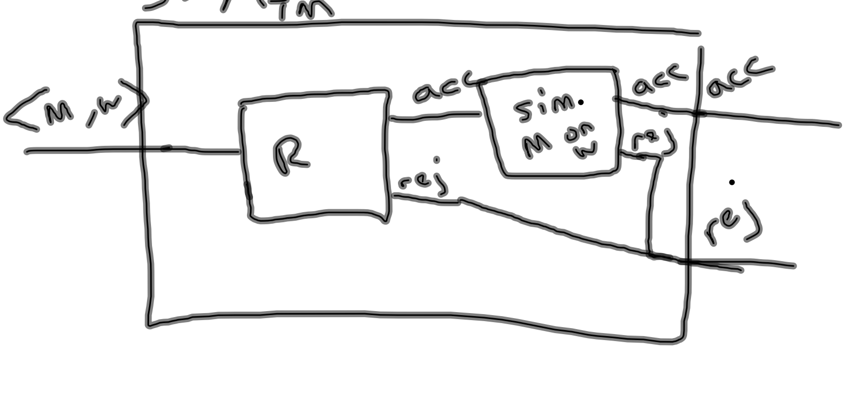
A_{TM}



S = on input $\langle M, w\rangle$

1. Run R on $\langle M, w\rangle$
2. If R rejects, reject
3. If R accepts, simulate M on w until M halts.
4. If M accepts, accept otherwise, reject

S: A_{TM}



$$E_{TM} = \{ \langle M \rangle \mid M \text{ is a TM and } L(M) = \emptyset \}$$

E_{TM} is undecidable

Proof (by contradiction)

Suppose it isn't. So TM R decides E_{TM}.

given M, w create TM M′

M′ = on input x

1. if $x \neq w$, reject.
2. if $x = w$, run M on w and accept if M accepts.

$$L(M') = \begin{cases} \{w\} & \text{if M accepts w} \\ \{\ \} & \text{otherwise.} \end{cases}$$

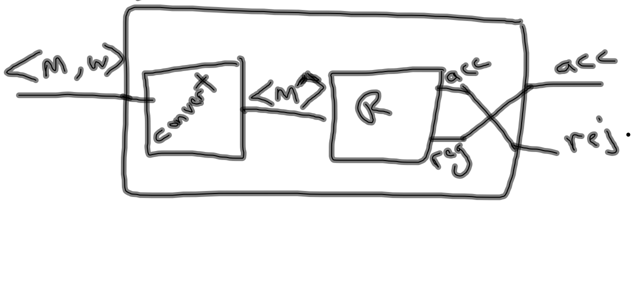
$L(M') \neq \emptyset$ iff M accepts w.

Construct S that decides A_{TM}.

S = on input $\langle M, w \rangle$

1. Use M, w to build M′
2. Run R on M′
3. if R accepts, reject.
   if R rejects, accept.

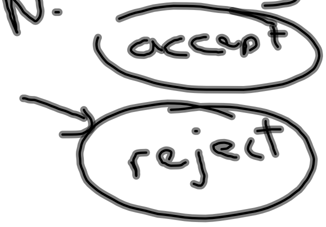
S.

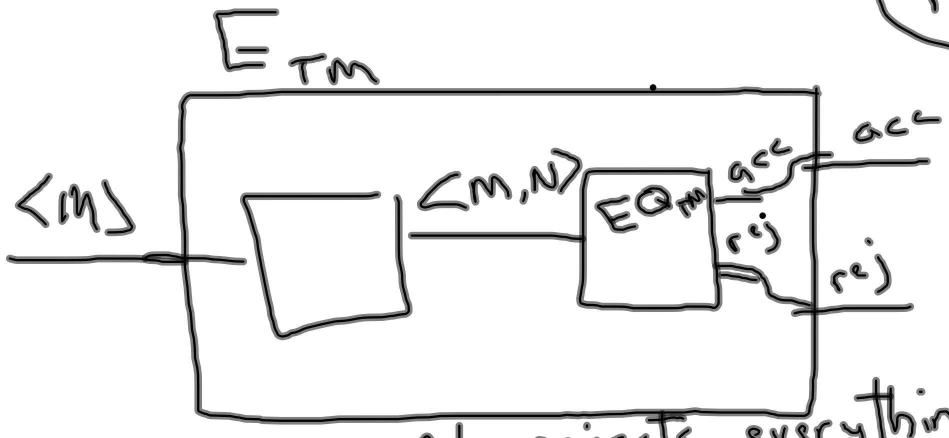


$EQ_{TM} = \{\langle M_1, M_2\rangle \mid$ TM's M_1 and M_2 and $L(M_1) = L(M_2)\}$

Reduction from E_{TM}

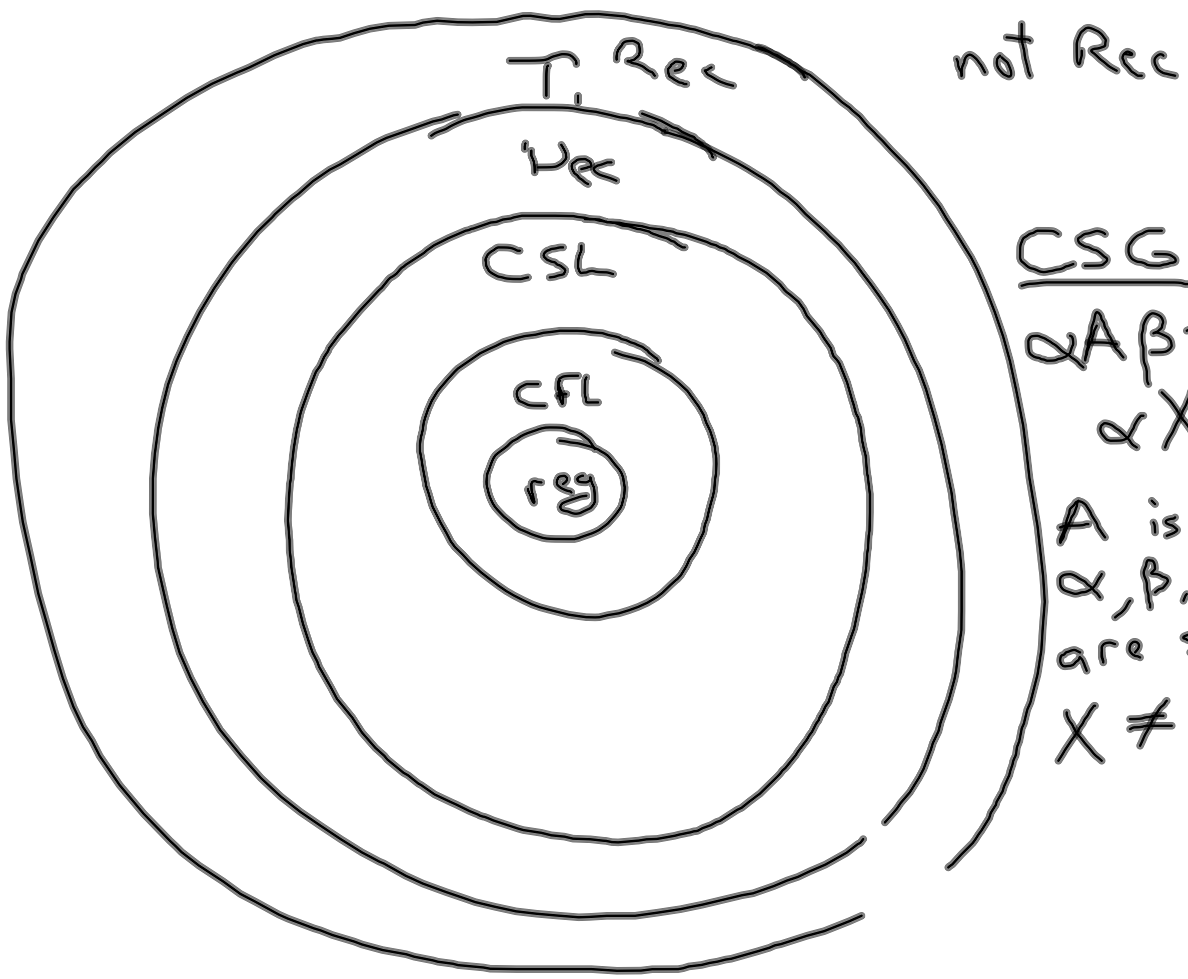
N:





TM N rejects everything.

$L(N) = \emptyset$



<u>CSG</u>

$\alpha A \beta \rightarrow \alpha X \beta$

A is a var
α, β, X are strings
$X \neq \varepsilon$