Some languages are <u>not</u> Turing recognizable.

<u>Proof</u>:

Set of all Turing Machines. is a subset of finite length strings so is countable.

Set of all languages is uncountable.

suppose it is countable

strings

	s_1	s_2	s_3	s_4	s_5	....
L_1	$\boxed{\checkmark}$	x	✓	✓	x	...
L_2	✓	$\boxed{\checkmark}$	x	✓	x	...
L_3	✓	x	$\boxed{\times}$	x	x	...
L_4	x	✓	✓	$\boxed{\times}$	x	...
⋮						

string s_5 is not in L_1

Construct L_n that is not in the list.

L_n's characteristic sequence

$s_i \in L_n$ if $s_i \notin L_i$

$s_i \notin L_n$ if $s_i \in L_i$

Therefore the set of lang. is uncountable.

Since TMs are countable and lang. are not, there must be some lang. not recognized by a TM.

$A_{TM} = \{\langle M, w\rangle \mid M$ is a TM that accepts $w\}$

Proof (by contradiction)

Suppose A_{TM} is decidable and H is TM that decides it.

$H(\langle M, w\rangle) =$ <u>accept</u> if M accepts w

<u>reject</u> if M does not accept w.

Construct TM D that uses H as a subroutine.

$D =$ on input $\langle M\rangle$ where M is a TM

1. Run H on input $\langle M, \langle M\rangle\rangle$
2. if H accepts $\rightarrow$ reject
   if H rejects $\rightarrow$ accept

Now run D on itself.

$$D(\langle D\rangle) = \begin{cases} \text{accept} & \text{if } D \text{ does not accept } \langle D\rangle \\ \text{reject} & \text{if } D \text{ accepts } \langle D\rangle \end{cases}$$

which is a contradiction.

Therefore, H cannot exist so A_{TM} is not decidable.

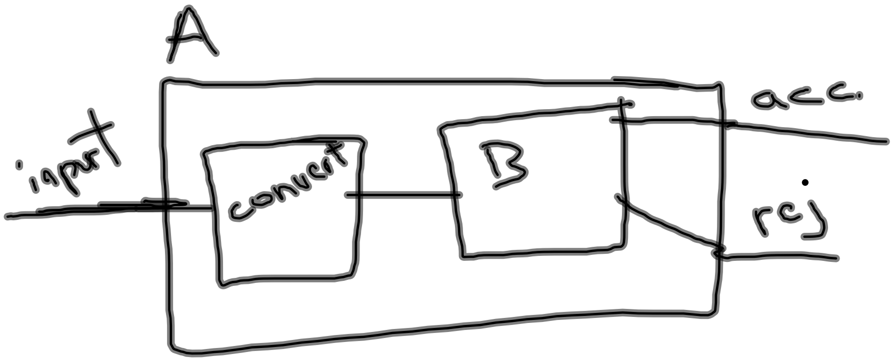
A language is co-Turing Recognizable if its complement is Turing Rec.

A language is decidable iff it is Turing Rec. and Co-Turing Rec.

$\overline{A_{TM}}$ is not Turing Rec.

Reducibility

A reduces to B



if A is reducible to B and B is decidable then A is decidable.

if A is reducible to B and A is undecidable, then B is undecidable.