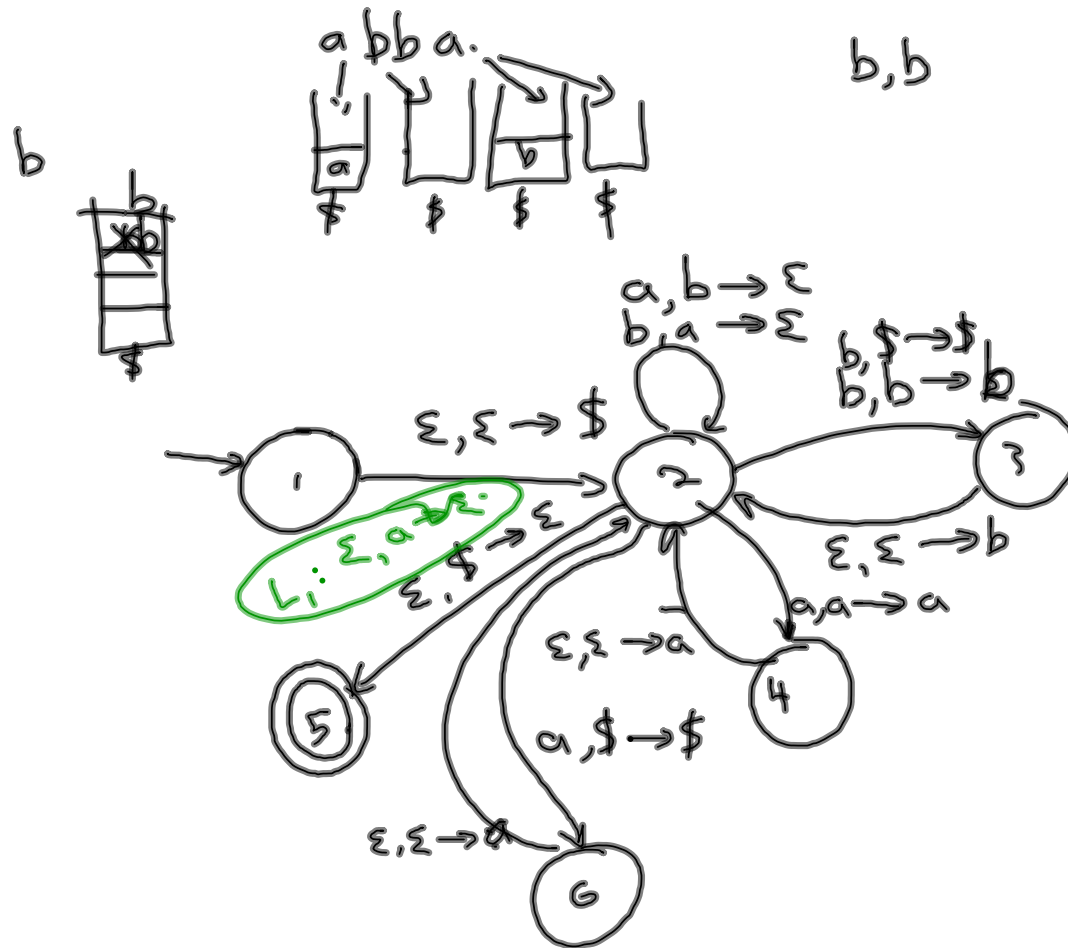


$$L_1 = \{ w \mid w \text{ has more } a\text{'s than } b\text{'s} \}$$

$$w \in \{a, b\}^*$$

$$L_2 = \{ w \mid w \in \{a, b\}^* \text{ } w \text{ has the same number of } a\text{'s as } b\text{'s} \}$$

- stack:



# Pumping Lemma for CFL's.

$$V \stackrel{*}{\Rightarrow} uVw \quad u, w: \text{strings of vars and terms.}$$
$$\stackrel{*}{\Rightarrow} uuVww$$

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If  $A$  is a context free language, then there is a number  $p$  (the pumping length) where, if  $s \in A$  of length at least  $p$ , then  $s$  may be divided into 5 pieces,

$s = uvxyz$  satisfying the conditions:

1. for each  $i \geq 0$   $uv^i xy^i z \in A$
2.  $|vy| > 0$
3.  $|vxy| \leq p$

$L = \{a^n b^n c^n \mid n \geq 0\}$  is not a CFL.

Proof by contradiction

Suppose  $L$  is context free.

Let  $p$  be the pumping length

Choose a string  $s = a^p b^p c^p$ .

So  $s$  can be split into 5 pieces,  
 $s = uvxyz$  according to the  
pumping lemma, because  $s \in L$  and  
 $|s| \geq p$ .

Since  $|vxy| \leq p$ , there can be  
at most 2 symbols in  $vxy$ .

Therefore  $uv^2xy^2z$  would have  
more of one or two symbols than  
the third so  $uv^2xy^2z \notin L$ ,  
which is a contradiction.

$\therefore L$  is not a CFL.