

CFG

$$\underline{A} \rightarrow w$$

$$w \in V \cup \Sigma$$

$$A \in V$$

Chomsky Normal Form
CFG (V, Σ, R, S)

$$A \rightarrow BC$$

$$A \rightarrow a$$

$$S \rightarrow \varepsilon$$

$$A, B, C, S \in V$$

$$a \in \Sigma$$

$$\begin{aligned} B &\neq S \\ C &\neq S \end{aligned}$$

Any CFL can be generated by a CFG in Chomsky N.F.

$$\text{CFG } G = (V, \Sigma, R, S)$$

construct $G' = (V', \Sigma, R', S')$
 s.t. G' is in Chomsky N.F.

1. create S' and rule $S' \rightarrow S$
2. remove $A \rightarrow \epsilon$

$$B \rightarrow uAw \in R \quad u, w: \text{strings of vars and terminals}$$

add $B \rightarrow uw \in R'$

$$B \rightarrow uAvAw \in R$$

$$\frac{B \rightarrow uvAw \mid uAvw \mid uvw \in R'}{B \rightarrow A \in R}$$

$$B \rightarrow \epsilon \in R'$$

3. Remove unit rules $A \rightarrow B$

for $A \rightarrow B$ and $B \rightarrow u$

$$A \rightarrow u \in R'$$

4. Replace $A \rightarrow s_1 s_2 s_3 \dots s_k$ s_i : var or terminal
 $k \geq 3$

create rules

$$A \rightarrow s_1 A_1$$

$$A_1 \rightarrow s_2 A_2$$

$$A_2 \rightarrow s_3 A_3$$

\vdots

$$A_{k-2} \rightarrow s_{k-1} s_k$$

If $s_i \in \Sigma$

replace $A_{i-1} \rightarrow s_i A_i$

w/ $A_{i-1} \rightarrow s_i A_i$

$$S_i \rightarrow s_i$$

$$R \quad \left. \begin{array}{l} A \rightarrow BAB \mid B \mid \epsilon \\ B \rightarrow OO \mid \epsilon \end{array} \right\}$$

$$R' \quad \begin{array}{l} \cancel{S' \rightarrow A} \\ \cancel{A \rightarrow BAB} \\ \cancel{A \rightarrow B} \\ \cancel{A \rightarrow \epsilon} \\ \cancel{B \rightarrow OO} \\ \cancel{B \rightarrow \epsilon} \\ S' \rightarrow \epsilon \\ A \rightarrow BB \\ A \rightarrow AB \\ A \rightarrow BA \\ \cancel{A \rightarrow B} \\ \cancel{A \rightarrow OO} \\ S' \rightarrow BS' \\ S' \rightarrow AB \\ S' \rightarrow VV \end{array}$$

$$\begin{array}{l} \cancel{S' \rightarrow BAB} \\ \cancel{S' \rightarrow BB} \\ \cancel{S' \rightarrow B} \\ \cancel{S' \rightarrow AB} \\ \cancel{S' \rightarrow BA} \\ \cancel{S' \rightarrow OO} \\ A \rightarrow BA' \\ A' \rightarrow AB \\ V \rightarrow O \\ B \rightarrow VV \\ A \rightarrow VV \end{array}$$

② remove $A \rightarrow \epsilon$
remove $B \rightarrow \epsilon$

~~$A \rightarrow A$~~

③ remove $S' \rightarrow A$
" $S' \rightarrow B$
" $A \rightarrow B$

④ remove $A \rightarrow BAB$
" $S' \rightarrow BAB$
" $B \rightarrow OO, S' \rightarrow OO$
 $A \rightarrow OO$

Linear Grammars

Rule RHS. has at most 1 var.

Regular Languages

