Proof of pumping lemma

Suppose $A$ is a regular language, a DFA $M \langle Q, \Sigma, s_0, F \rangle$ recognizes $A$.

Let $p$ = number of states of $M$
Let $s = s_1s_2\ldots s_n$ be a string in $A$ where $n \geq p$.

Let $r_1, \ldots, r_{n+1}$ be a sequence of states that $M$ enters while processing $s$. So $r_{i+1} = S(r_i, s_i)$ for $1 \leq i \leq n$.

Among the first $p+1$ states in the seq., two must be the same state.

Call those states $r_j$ and $r_l$.

$j < l \leq p+1$

Let $x = s_1 \ldots s_{j-1}$
$y = s_j \ldots s_{l-1}$
$z = s_l \ldots s_n$

1. For each $i \geq 0$, $xy^iz \in A$

   states $r_j, \ldots r_{l-1}$ can be repeated or skipped.

2. $|y| > 0$
3. $|xy| \leq p$
Prove lang. A is not regular.

1. Assume A is regular
2. the pumping lemma must hold.
   so there must be a length, \( p \)
   for which all strings \( s, \ |s| \geq p \)
   can be "pumped."
3. find a string \( s \in A, \ |s| \geq p \)
   that can't be.
   - consider all ways the string can
     be divided into \( xyz \).
     show \( xy^iz \notin A \) for some \( i \)
4. conclude A is not regular.
Prove $B = \{0^n1^n | n \geq 0 \}$ is not regular.

Suppose $B$ is regular.
Let $p$ be the pumping length given in the pumping lemma.
Choose $s = 0^p1^p$ since $s \in B$ and $|s| > p$.
by the pumping lemma, we can split $s$ into 3 pieces $s = xyz$.

3 cases
1. $y$ is all 0's
   consider $x|yy|yz \notin B$ since it has more 0's than 1's
2. $y$ is all 1's
3. $y$ contains 0's and 1's
   $x|yy|yz$ 0's and 1's out of order.

$\therefore B$ is not regular.
\[ C = \{ w \mid w \text{ has an equal number of 0's and 1's} \} \]

Show \( C \) is not regular.

Let \( s = 0^p1^p \)
\[ s \in C, \ |s| > p \]
\[ s = xyz \]
since \( |xy| \leq p \), \( y \) must be all 0's.

Suppose \( C \) is regular

Regular lang. are closed under \( \cap \)

consider \( C \cap 0^*1^* = 0^n1^n \)

which is not regular.

\( \therefore C \) is not regular.
\[ D = \{ \text{ww}^p \mid w \in \{0,1\}^* \} \]
\[ S = 0^p 110^p \]