Proof of pumping lemma

Suppose A is a regular lang.
a DFA $M(Q, \Sigma, \delta, q_0, F)$
recognizes A.
Let $p =$ number of states of M
Let $s = s_1 s_2 \dots s_n$ be a string in A

where $n \geq p$.

Let $r_1, \dots r_{n+1}$ be a sequence of states that M enters while processing s. So $r_{i+1} = \delta(r_i, s_i)$ for $1 \leq i \leq n$

Among the first $p+1$ states in the seq., two must be the same state.

Call those states r_j and r_ℓ.

$j < \ell \quad \ell \leq p+1$

Let $x = s_1 \dots s_{j-1}$

$y = s_j \dots s_{\ell-1}$

$z = s_\ell \dots s_n$

1. For each $i \geq 0 \quad xy^i z \in A$

states $r_j \dots r_{\ell-1}$ can be repeated or skipped.

2. $|y| > 0$
3. $|xy| \leq p$

Prove lang. A is not regular.

1. Assume A is regular
2. the pumping lemma must hold.
   so there must be a length, p
   for which all strings s, $|s| \geq p$
   can be "pumped."
3. find a string $s \in A$, $|s| \geq p$
   that can't be.

   - consider all ways the string can
     be divided into xyz.
     show $xy^iz \notin A$ for some i
4. conclude A is not regular.

Prove $B = \{0^n 1^n \mid n \geq 0\}$ is not regular.

Suppose B is regular.
Let p be the pumping length given in the pumping lemma.
Choose $s = 0^p 1^p$
since $s \in B$ and $|s| > p$
by the pumping lemma, we can split s into 3 pieces
$s = xyz$

3 cases
1. y is all 0's
   consider $xyyz$
   $xyyz \notin B$ since it has more 0's than 1's

2. y is all 1's
3. y contains 0's and 1's
   $xyyz$ 0's and 1's out of order.

$\therefore B$ is not regular.

$\times$

$C = \{ w \mid w$ has an equal number of 0's and 1's$\}$

Show C is not regular.

Let $s = 0^p 1^p$

$s \in C$, $|s| > p$

$s = xyz$

since $|xy| \leq p$, y must be all 0's.

Suppose C is regular

Regular lang. are closed under $\cap$

consider $C \cap 0^* 1^* = 0^n 1^n$

which is not regular.

$\therefore$ C is not regular.

$D = \{ww^R \mid w \in \{0,1\}^*\}$

$s = 0^p 1 1 0^p$