$$M = (Q, \Sigma, \delta, q_0, f)$$

M recognizes a language L

if $L = \{ w \mid M \text{ accepts } w \}$

Regular language

Operations

A, B : languages

regular operations

$$A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$$

$$A \circ B = \{ xy \mid x \in A \text{ and } y \in B \}$$

$$A^* = \{ x_1 x_2 x_3 \cdots x_k \mid k \geq 0 \text{ and each } x_i \in A \}$$

$\Sigma = \{a, b\}$

$A = \{aa, a, b, bb\}$

$B = \{ab, a, b, ba\}$

$C = \{a, b\}$

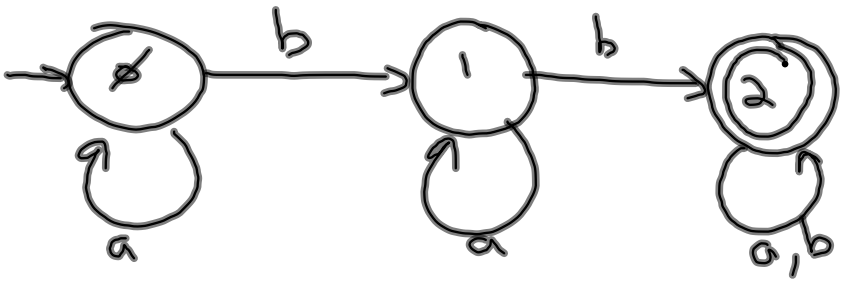
$A \cup B = \{aa, ab, a, b, ba, bb\}$

$A \circ C = \{aaa, aab, aa, ab, ba, bb, bba, bbb\}$

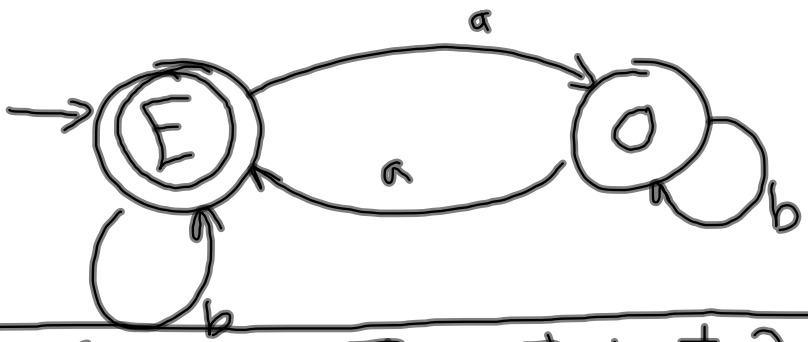
$C^* = \{\varepsilon, a, b, aa, ab, ba, bb, aaa, aab, aba, abb, \cdots\}$

Σ^* : all strings over the alphabet Σ.

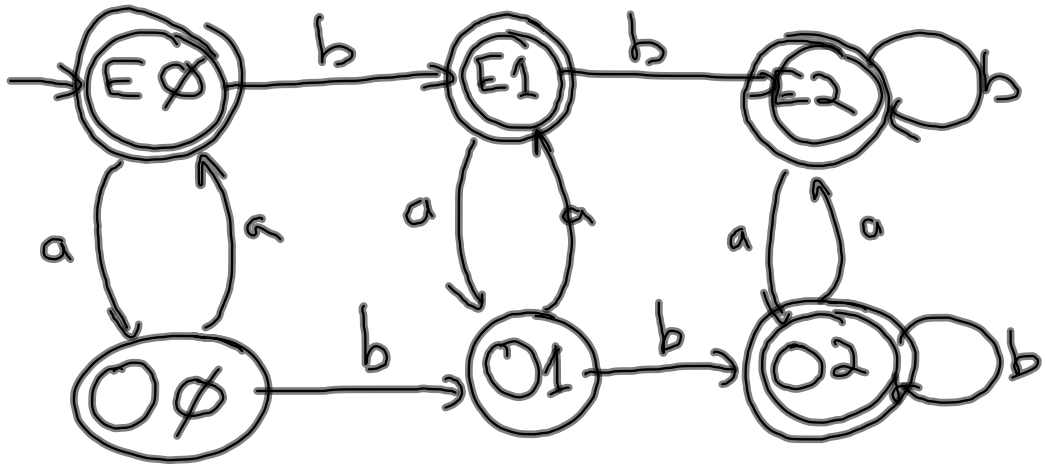
contains at least 2 b's



even # of a's



$L = \{ w \mid w$ contains at least 2 b's or an even number of a's $\}$


E∅
E1
E2
O∅
O1
O2
a
b

Regular lang. are closed under union

A, B : regular lang.
show $A \cup B$ is regular

Let M_1 recognize A
" M_2 " B

Build M that recognizes $A \cup B$

$M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$
$M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$
construct $M = (Q, \Sigma, \delta, q_0, F)$

1. Q: $Q_1 \times Q_2 = \{ (r_1, r_2) \mid r_1 \in Q_1 \text{ and } r_2 \in Q_2 \}$

2. Σ

4. $q_0 = (q_1, q_2)$
5. $F = \{ (r_1, r_2) \mid r_1 \in F_1 \text{ or } r_2 \in F_2 \}$

3. δ: for each $(r_1, r_2) \in Q$ and $a \in \Sigma$

$$\delta((r_1, r_2), a) = (\delta_1(r_1, a), \delta_2(r_2, a))$$