\( M = (Q, \Sigma, \delta, q_0, f) \)

\( M \) recognizes a language \( L \)

\[ \text{if } L = \{ \omega \mid M \text{ accepts } \omega \} \]

Regular language
Operations

$A, B$: languages

$A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$

$A \cdot B = \{ xy \mid x \in A \text{ and } y \in B \}$

$A^* = \{ x_1 x_2 \ldots x_k \mid k \geq 0 \text{ and each } x_i \in A \}$

$\Sigma = \{a, b\}$

$A = \{aa, a, b, bb\}$

$B = \{ab, a, b, ba\}$

$C = \{a, b\}$

$A \cup B = \{aa, ab, a, b, ba, bb\}$

$A \cdot C = \{aaa, aab, aa, ab, ba, bb, bba, bbb\}$

$C^* = \{ \varepsilon, a, b, aa, ab, ba, bb, aab, aba, abb, \ldots \}$

$\Sigma^*: \text{all strings over the alphabet } \Sigma$. 
$L = \sum w \mid w \text{ contains at least 2 } b \text{'s or an even number of } a \text{'s}$
Regular lang. are closed under union

A, B: regular lang.
show AUB is regular

Let M₁ recognize A
" M₂ " B

Build M that recognizes AUB

\[ M₁ = (Q₁, \Sigma, \delta₁, q₀₁, F₁) \]
\[ M₂ = (Q₂, \Sigma, \delta₂, q₀₂, F₂) \]

construct \( M = (Q, \Sigma, \delta, q₀, F) \)

1. \( Q: Q_1 \times Q_2 = \{ (r₁, r₂) \mid r₁ \in Q₁ \text{ and } r₂ \in Q₂ \} \)

2. \( \Sigma \)

4. \( q₀ = (q₀₁, q₀₂) \)

5. \( F = \{ (r₁, r₂) \mid r₁ \in F₁ \text{ or } r₂ \in F₂ \} \)

3. \( \delta: \) for each \( (r₁, r₂) \in Q \)
and \( a \in \Sigma \)
\[ \delta((r₁, r₂), a) = \]
\[ (\delta₁(r₁, a), \delta₂(r₂, a)) \]