\[ EQ_{TM} = \{ <m_1, m_2> \mid m_1, m_2 \text{ are TM and } L(m_1) = L(m_2) \} \]

\[ EQ_{TM} \text{ is undecidable} \]

proof by contradiction:
assume \( EQ_{TM} \) is decidable
by \( TM, R \).
Show that \( E_{TM} \) is decidable
(which is a contradiction)

Construct \( S \): on input \( <m> \)

1. Run \( R \) on input \( <m, m> \)
   where \( m \) is a TM that
   rejects all input.

2. If \( R \) accepts, accept.
   If \( R \) rejects, reject.

So \( S \) decides \( E_{TM} \) which
is a contradiction.

Therefore, there is no decider
for \( EQ_{TM} \).
Configurations
101100g, 10110

Computational history
Sequence of configs
C₁, C₂, C₃, ... Cₗ

Accepting comp. history
a) C₁ is start config.
b) Cₗ is an accept config.
c) each Cᵢ follows from Cᵢ₋₁ according to rules for M.
Linear Bounded Automata (LBA)
- like a TM
- tape is only as long as the input.

LBA's accept CSLs

can decide \( A_{DFA}, A_{CFG}, E_{DFA}, E_{CFG} \)

\( M \) is a LBA

\( q \) states
\( \sigma \) symbols
\( n \) input length

There are exactly \( q \cdot n \cdot \sigma \)

distinct configurations

\( A_{LBA} \) is decidable
\( E_{LBA} \) is undecidable
\[ \log_2 101 \rightarrow 100_2 \rightarrow \log_2 101 \]
Post Correspondence Problem (PCP)

given

\[
\begin{align*}
\left[ \frac{b}{ca} \right] & , \left[ \frac{a}{ab} \right] , \left[ \frac{ca}{a} \right] , \left[ \frac{abc}{c} \right] \\
\left[ \frac{a}{ab} \right] & , \left[ \frac{b}{ca} \right] , \left[ \frac{ca}{a} \right] , \left[ \frac{a}{ab} \right] , \left[ \frac{abc}{c} \right] \\
\end{align*}
\]

\[abc \cdot a \cdot a \cdot abc\]