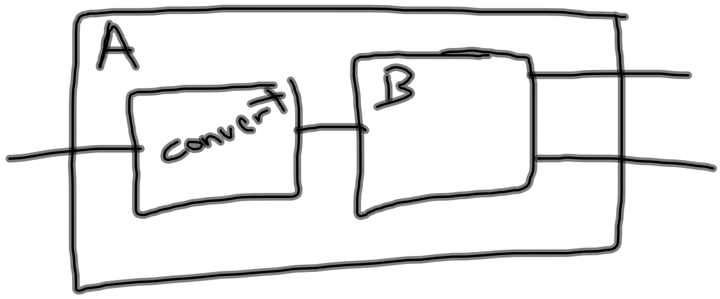
Problem Reduction

Convert one problem to another

A reduces to B
- we can use solution of B to solve A



if A is reducible to B and B is decidable, then A is decidable.

if A is undecidable and reducible to B then B is undecidable.

$HALT_{TM} = \{\langle M, w\rangle \mid M \text{ is a TM that halts on } w\}$

$HALT_{TM}$ is undecidable

proof by contradiction

Suppose $HALT_{TM}$ is decidable w/ TM R

Construct a TM S to decide A_{TM} using R.

S = on input $\langle M, w\rangle$

1. Run R on input $\langle M, w\rangle$.
2. If R rejects, reject
3. If R accepts, we know M accepts w or M rejects w.
   - Simulate M on w, If M accepts, accept otherwise reject.

So S decides A_{TM}.

However, A_{TM} is undecidable so R does not exist. $\times\!\!\times$

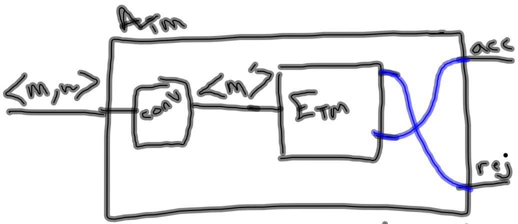
Reduced A_{TM} to $Halt_{TM}$

$E_{TM} = \{ \langle M \rangle \mid M$ is a TM and $L(M) = \emptyset \}$

E_{TM} is undecidable by reduction from A_{TM}

Proof by contradiction

Suppose E_{TM} is decidable by T.M. R.



set up: M' is not empty iff M accepts w.

Given M, w create M'

M' on input x

1. if $x \neq w$ reject
2. if $x = w$ run M on w and accept if M accepts.

Construct S to decide A_{TM}

S on input $\langle M, w \rangle$

1. use description of M and w to build M' (as above).
2. run R on $\langle M' \rangle$.
3. if R accepts, reject. if R rejects, accept.