NP-Complete (B)

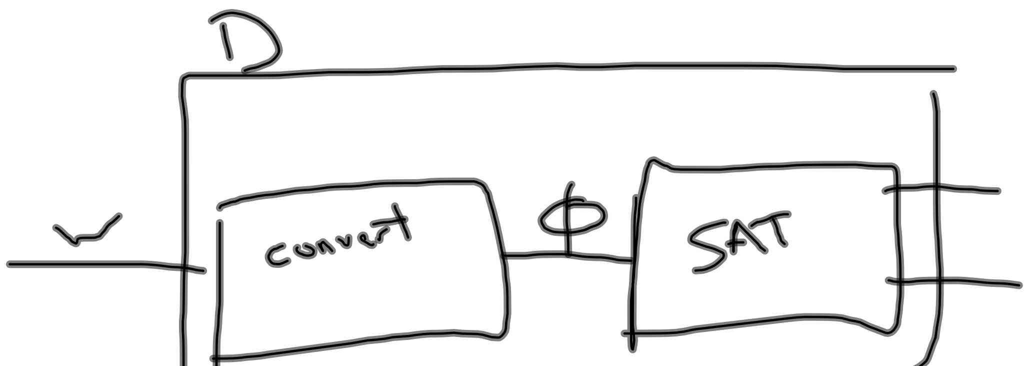
1. $B \in NP$
2. Every $A \in NP$ is poly. time reducible to B ($A \leq_p B$) (NP-Hard)

SAT is NP-Complete

$\forall A \in NP, \; A \leq_p SAT$

Suppose A is in NP

A is decided by a non-det poly-time TM; D

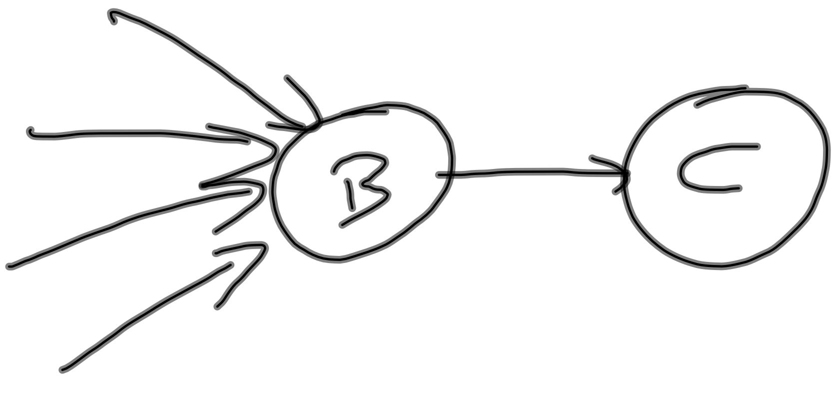


Φ is satisfiable iff w is accepted by D.

If B is NPC and

$B \leq_p C$, for $C \in NP$,

then C is NPC.



3-SAT is NP complete.

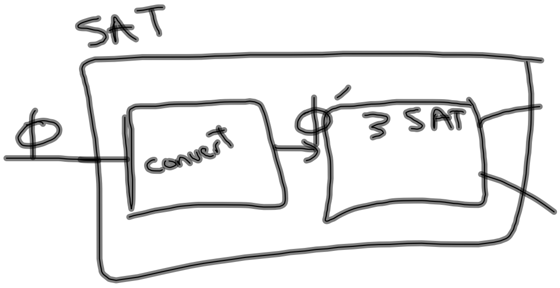
a. Is 3-SAT NP?

<u>NP algorithm</u>

- non-deterministically generate all assignments to all variables
- evaluate the formula for the assignments
- if evaluation is true, accept
  else reject

b. 3-SAT is NP-Hard

SAT $\le_p$ 3-SAT



1. Convert ϕ to CNF
2. Convert ϕ so that all terms have exactly 3 literals.

terms in ϕ	terms in ϕ'
3 literals	$\longrightarrow$ copy
1 literal (x)	$(x \vee x \vee x)$
2 literals $(x \vee y)$	$(x \vee y \vee y)$
4 $(x_1 \vee x_2 \vee x_3 \vee x_4)$	$(x_1 \vee x_2 \vee a) \wedge (\bar{a} \vee x_3 \vee x_4)$
5 $(x_1 \vee x_2 \vee x_3 \vee x_4 \vee x_5)$	$(x_1 \vee x_2 \vee a) \wedge (\bar{a} \vee x_3 \vee b) \wedge (\bar{b} \vee x_4 \vee x_5)$

- 3-SAT is NPC
- CLIQUE is NPC

  3-SAT $\le_p$ CLIQUE

  CLIQUE is in NP.