3-SAT

$$(x_1 \vee x_2 \vee x_3) \wedge (\overline{x_2} \vee x_1 \vee x_4) \wedge (\quad)$$

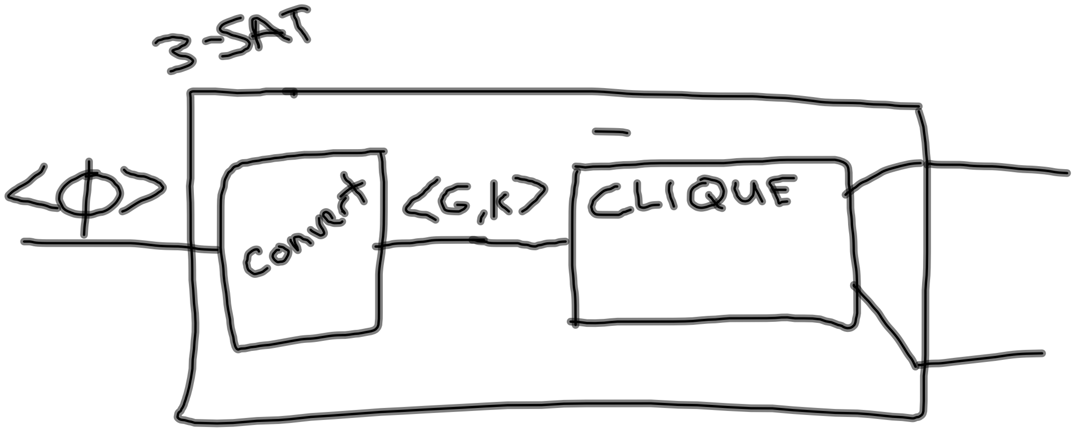
3 CNF

$$3SAT = \{\langle\phi\rangle \mid \phi \text{ is a satisfiable 3cnf formula}\}$$

3-SAT is poly. time reducible to CLIQUE

$$\text{3-SAT} \leq_p \text{CLIQUE}$$

$\text{CLIQUE} = \{\langle G, k\rangle \mid G \text{ is an undir. graph w/ a clique of size } k\}$



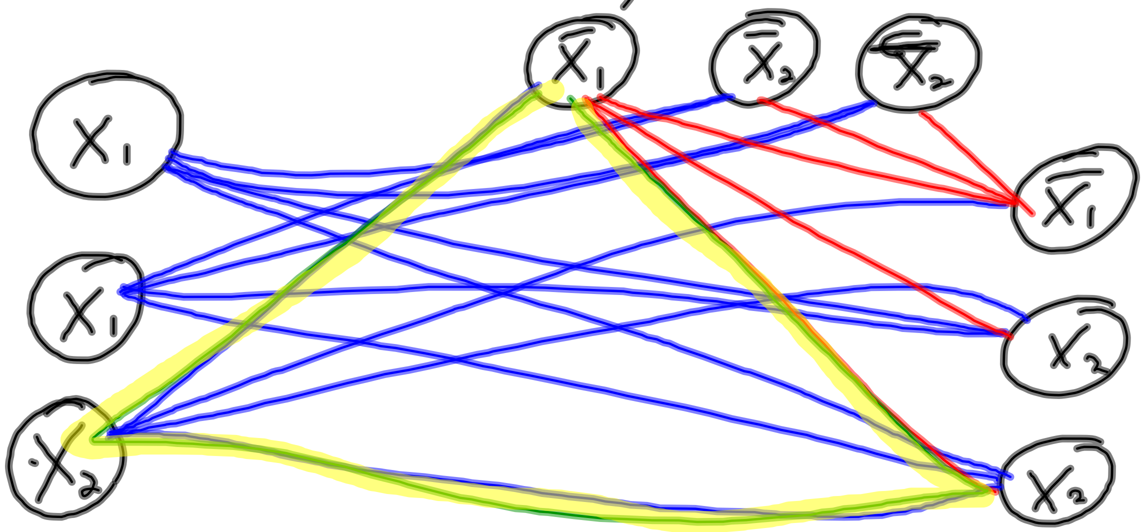
$$\phi = (a_1 \vee b_1 \vee c_1) \wedge (a_2 \vee b_2 \vee c_2) \wedge \dots$$
$$\wedge (a_k \vee b_k \vee c_k)$$

construct <G, k>

- graph has k groups of 3 nodes each

- edges in G between all pairs of nodes except
  1. nodes in the same group
  2. between nodes w/ contradictory labels e.g. x and $\overline{x}$

$$\Phi = (x_1 \vee x_1 \vee x_2) \wedge$$
$$(\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_2) \wedge$$
$$(\bar{x}_1 \vee x_2 \vee x_2)$$



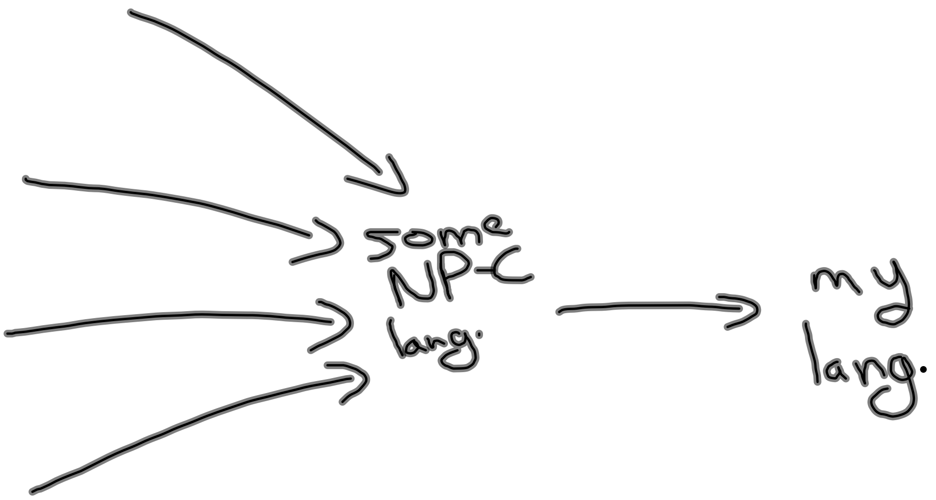
Φ is satisfiable iff
G has a k-clique.

I. suppose Φ is satisfiable
- each term has at least one true literal.
- select the nodes that represent ~~that~~ those literal
- the selected nodes form a k-clique

II. Suppose G has a k-clique
- no two node of the clique occur in the same group
- each of the k groups contains exactly one node of the k-clique.
- assign values to the variables so the literal labelling the node is true. e.g. $\bar{x} \Longrightarrow x := f$
- that assignment satisfies Φ

A language B is NP Complete if it satisfies two conditions:

1. B is in NP.
2. every language A in NP is polynomial time reducible to B.



If B is NP-C and
B is in P then $P = NP$.

If B is in NPC and
$B \leq_p C$ for C in NP,
then C is NP-C.

Cook-Levin Thm.: SAT is
NP-complete.

Suppose A is in NP.

so A is decided by

a Non-det TM in poly-time

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