$A_{TM} = \{ \langle M, w \rangle \mid M$ is a TM that accepts $w \}$

A_{TM} is not decidable.

Proof by contradiction

Suppose A_{TM} is decidable and H is the TM that decides it.

$$H(\langle M, w \rangle) = \begin{cases} \text{accept} & \text{when } M \text{ accepts } w \\ \text{reject} & \text{when } M \text{ does not accept } w \end{cases}$$

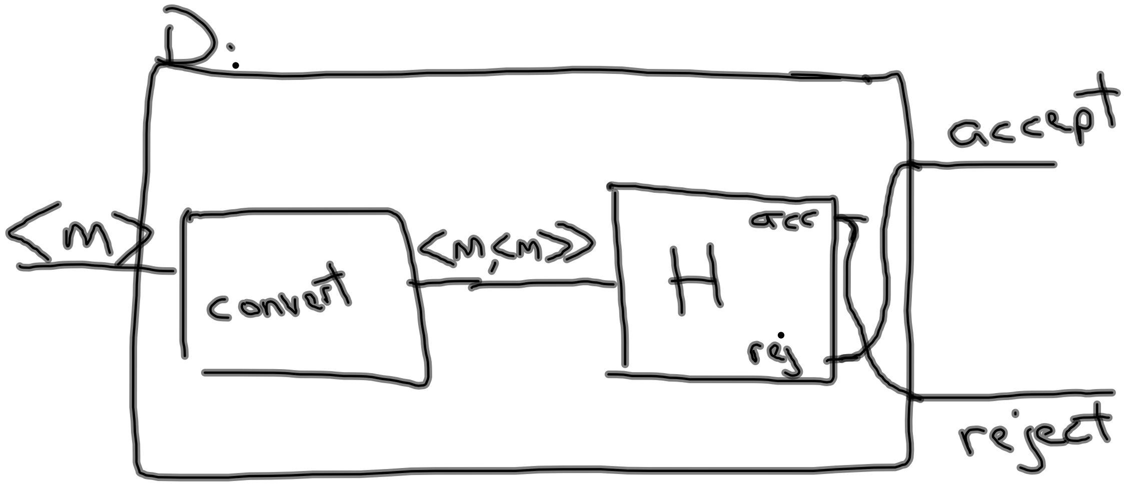
Construct D, a T.M., that uses TM H.

D: on input $\langle M \rangle$ where M is a TM

1. run H on input $\langle M, \langle M \rangle \rangle$
2. output the opposite of H.

   if H accepts $\rightarrow$ D reject

   if H rejects $\rightarrow$ D accept



run D on input <D>

$$D(\langle D\rangle) = \begin{cases} \text{accepts} & \text{if } D \text{ does not accept } \langle D\rangle \\ \text{rejects} & \text{if } D \text{ accepts } \langle D\rangle \end{cases}$$

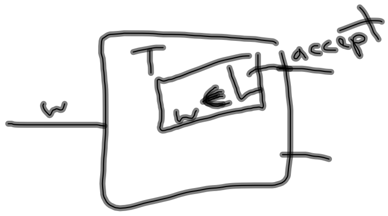
a contradiction.

So H cannot exist.

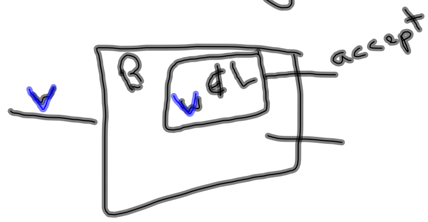
$\therefore$ A_{TM} is not decidable.

co-Turing recognizable
if its complement is
Turing recognizable.

L is recognizable
T rec. L



L is co-Turing rec., R. TM

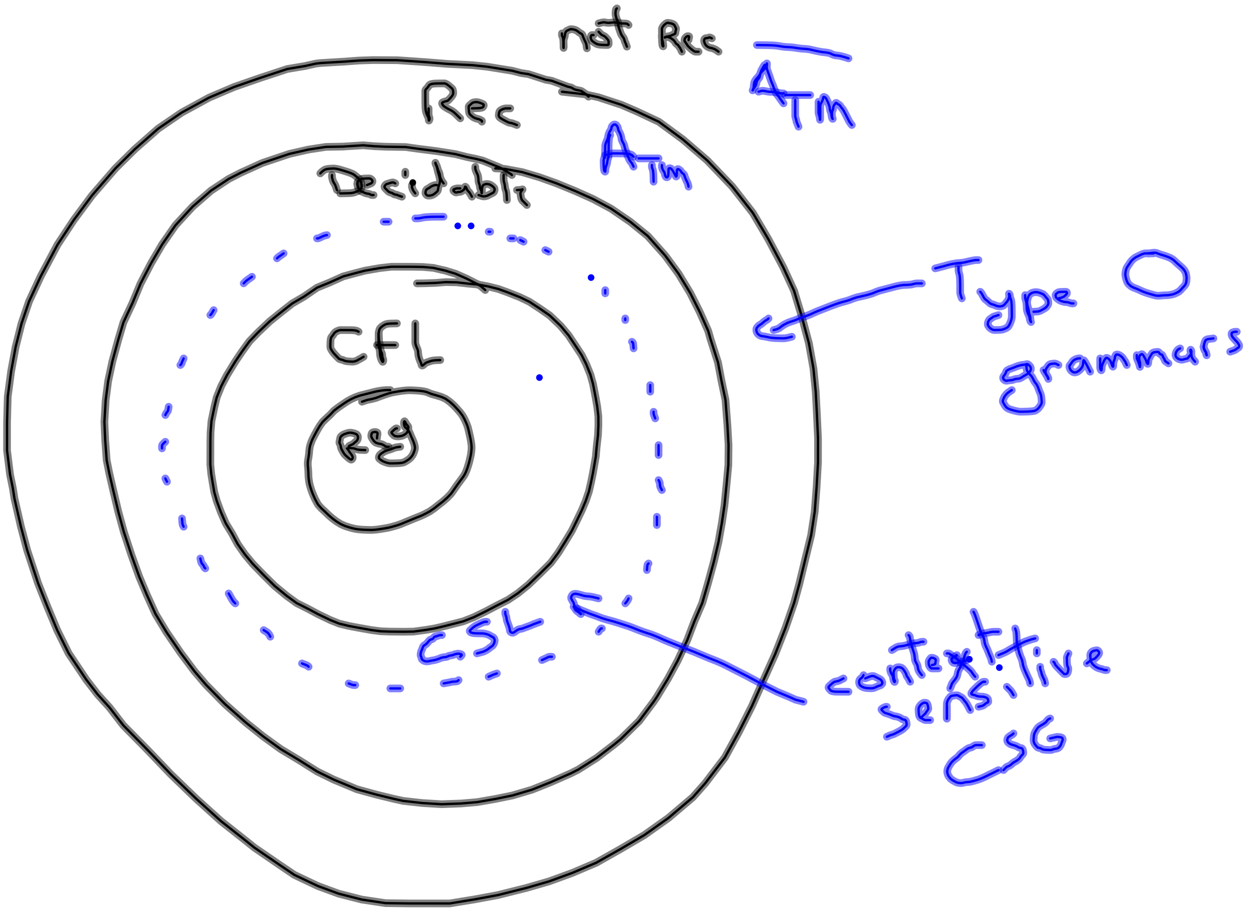


a language is decidable iff
it is Turing rec. <u>and</u>
co-Turing rec.

L is recog.
and $\overline{L}$ is recog.

A_{TM} is not decidable
A_{TM} is Turing recognizable.

A_{TM} is <u>not</u> co-Turing recog.
so $\overline{A_{TM}}$ is not Turing recog


not Rec
$\overline{A_{TM}}$
Rec
A_{TM}
Decidable
CFL
Reg
CSL
Type 0 grammars
context sensitive
CSG

Rules

$S \rightarrow aSa$

CFG:
LHS: 1 variable
RHS: string of vars and term.

$\alpha A \beta \rightarrow \alpha X \beta$

CSG: $X \neq \varepsilon$
α, β, X are strings of vars and terms
A: variable

$L = \{ a^n b^n c^n \mid n > 1 \}$

$S \rightarrow S_a BCS_1 \mid S_a BC$

$S_1 \rightarrow ABCS_1 \mid ABC$

$BA \rightarrow AB$
$CA \rightarrow AC$
$CB \rightarrow BC$
$S_a \rightarrow a$

$aA \rightarrow aa$
$aB \rightarrow ab$
$bB \rightarrow bb$
$bC \rightarrow bc$
$cC \rightarrow cc$

S
$S_a BCS_1$
$a BC\underline{S_1}$
$aB\underline{CA}BC$
$a\underline{BA}CBC$
$\underline{aA}BCBC$
$aaB\underline{CB}C$
$a\underline{aB}BCC$
$aabBCC$
$\vdots$
$aabbcc$