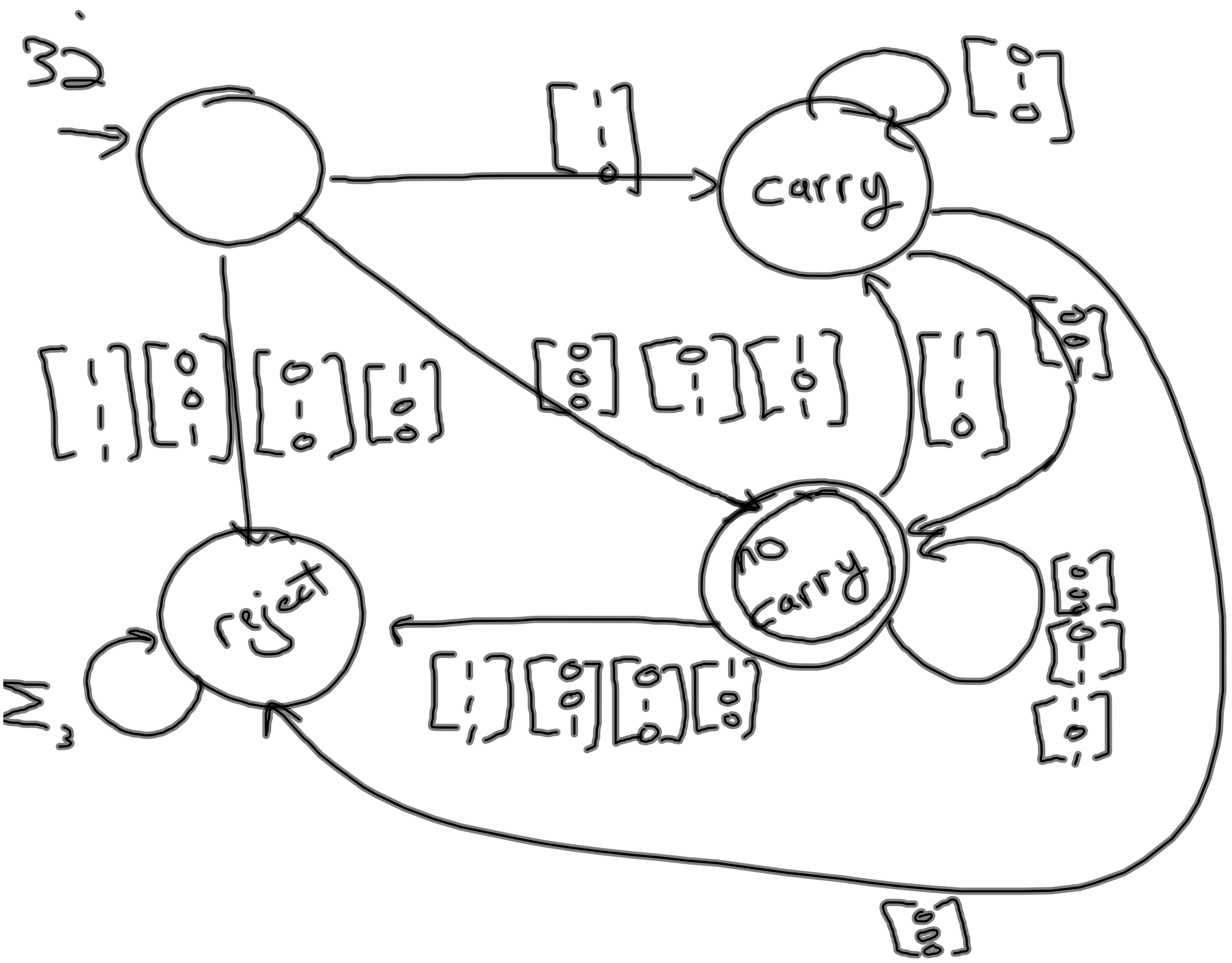
$$\begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix} \cdots \begin{bmatrix}1\\1\\1\end{bmatrix}$$

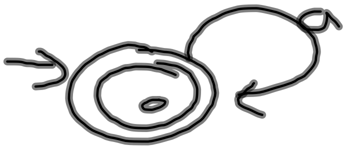
$$\begin{bmatrix}0\\0\\1\end{bmatrix} \begin{bmatrix}1\\1\\0\end{bmatrix} \begin{bmatrix}0\\1\\1\end{bmatrix} \in B$$

$$\begin{array}{r} {\scriptstyle 1} \\ 010 \\ 011 \\ \hline 101 \end{array}$$

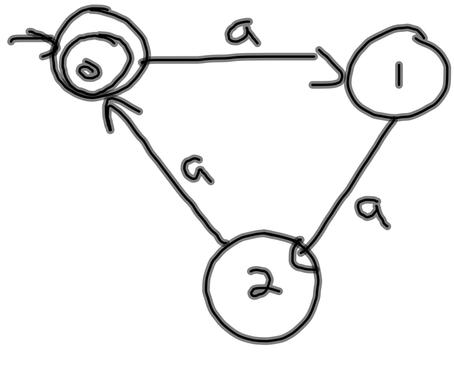


36.

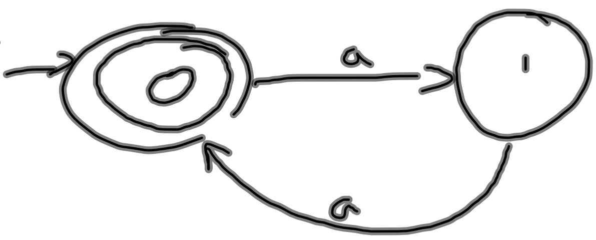
B_1



B_3



B_2



B_n:

$Q = \{0, \dots (n-1)\}$

$\Sigma = \{a\}$

$\delta(q_0, a) = (q_0+1)\%n$

$q_0 = 0$

$F = \{0\}$

Reg. exp

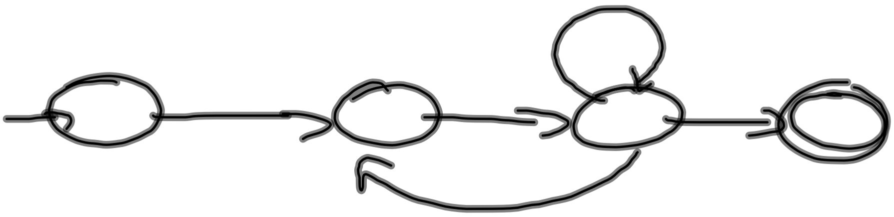
$B_n = L((a^n)^*)$

If A is a regular language, then there is a number p (the pumping length) where, if s is any string in A of length at least p, then s may be divided into 3 pieces s=xyz satisfying the following conditions:

1. for each $i >= 0$ xy^iz is in A
2. $|y| > 0$
3. $|xy| <= p$

Idea: DFA M rec. A

let p = # of states in M



Let $M = (Q, \Sigma, \delta, q_0, F)$
be a DFA recognizing A.
and p is the number of states
in M.

Let $s = s_1 s_2 s_3 \ldots s_n$ be a string
in A of length n, where $n \geq p$

Let $r_1, r_2, \ldots, r_{n+1}$ be a sequence
of states that M enters while
processing s, so $r_{i+1} = \delta(r_i, s_i)$
for $1 \leq i \leq n$.

This sequence has length $n+1$ which
is at least $p+1$.

Among the first $p+1$ elements,
two must be the same state
call them r_j and r_ℓ
r_j is first occ. r_ℓ is second
$\ell \leq p+1$

Now let $x = s_1 \cdots s_{j-1}$
$y = s_j \ldots s_{\ell-1}$
$z = s_\ell \ldots s_n$

Consider the conditions
1. for $i \geq 0$ $xy^i z \in A$ ✓
$\therefore$ 2. $|y| > 0$
3. $|xy| \leq p$ $\ell \leq p+1$

QED $\square$