$B_1$  

$B_2$  

Reg. exp  

$B_0 = L((a^*)^*)$  

$B_n$  

$Q = \{q_0, \ldots, (n-1)\}$  

$\Sigma = \{a, \bar{a}\}$  

$\delta(q_0, a) = (q_0) \% n$  

$q_0 = 0$  

$F = \{0\}$
If \( A \) is a regular language, then there is a number \( p \) (the pumping length) where, if \( s \) is any string in \( A \) of length at least \( p \), then \( s \) may be divided into 3 pieces \( s=xyz \) satisfying the following conditions:

1. for each \( i \geq 0 \) \( xy^iz \) is in \( A \)
2. \( |y| > 0 \)
3. \( |xy| \leq p \)

\[
\text{Idea: DFA } M \text{ rec. } A \\
\text{let } p = \# \text{ states in } M
\]
Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA recognizing $A$, and $p$ is the number of states in $M$.

Let $s = s_1 s_2 s_3 \ldots s_n$ be a string in $A$ of length $n$, where $n \geq p$.

Let $r_1, r_2, \ldots, r_{n+1}$ be a sequence of states that $M$ enters while processing $s$, so $r_{i+1} = S(r_i, s_i)$ for $1 \leq i \leq n$.

This sequence has length $n+1$ which is at least $p+1$.

Among the first $p+1$ elements, two must be the same state.

Call them $r_j$ and $r_k$.

$r_j$ is first $\omega$, $r_k$ is second

\[ l \leq p+1 \]

Now let $x = s_1 \ldots s_{j-1}$

$y = s_j \ldots s_{k-1}$

$z = s_k \ldots s_n$

Consider the conditions:

1. for $i = 0$ $xy'z \in A$

2. $|y| > 0$

3. $|xy| \leq p$ \( l \leq p+1 \)

\[ \text{QED} \]