Pumping lemma for CFL's

If A is a CFL, then there is some number p (the pumping length) where, if s is a string in A of length at least p, then s may be divided into 5 pieces $s = uvxyz$ satisfying the conditions:

1. for each $i \geq 0$
   $$uv^ixy^iz \in A$$

2. $|vy| > 0$

3. $|vxy| \leq p$
$$B = \{ a^n b^n c^n \mid n \geq 0 \}$$
is not context-free.

Assume $B$ is context-free
Let $p = \text{pumping length}$
Choose $s \in B$

$$s = a^p b^p c^p$$

**Case 1:** $vxy$ all one letter

$$uv^2xy^2z \text{ has more of that letter}$$

**Case 2:** $v$ and $y$ are each made up of a single letter

$$uv^2xy^2z \text{ has more of two of the letters}$$

**Case 3:** $v$ or $y$ contain multiple different symbols

$$uv^2xy^2z \not\in B, \text{ bad ordering}$$

\[ \text{aaa} \text{hhh} \text{bccc} \]

\[ uuxyzz \]
\[ D = \{ ww \mid w \in \{0,1\}^* \} \]

is not a CFL

Choose \( s = 0^p 1^p 0^p 1^p \).

Split \( s = uvxyz \) where \( |vxy| \leq p \)

- \( vxy \) must cross the center of \( s \)

- so \( vxy \) is some 1's followed by some 0's

\[ uv^2xy^2z \]

will have too many 1's in the first part or too many 0's in the second
$L = \text{palindromes that contain an equal number of 0's and 1's}$

$X \ 011010110$
$X \ 0110110110$
$\checkmark \ 01100110$

$S = O^p \mid \mid O^p$

1. $vxy$ is all 0's (from same $O^p$) more 0's than 1's

2. $vxy$ is all 1's more 1's than 0's

3. $vxy$ are 0's and 1's no longer a palindrome

$L_1: \text{palindromes}$
$L_2: \text{equal 0's and 1's}$

$L = L \cap L_2$

CFL's not closed under $\cap$
Turing machines

Alan Turing

Alonzo Church

Church-Turing Thesis

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