
CFL
Regular
Lang

Pumping lemma for CFL's

If A is a CFL, then there is some number p (the pumping length) where, if s is a string in A of length at least p, then s may be divided into 5 pieces $s = uvxyz$ satisfying the conditions:

1. for each $i \geq 0$
   $$uv^i x y^i z \in A$$

2. $|vy| > 0$

3. $|vxy| \leq p$

$$B = \{ a^n b^n c^n \mid n \geq 0 \}$$

is not context free.

Assume B is context free

Let p = pumping length

Choose $s \in B$

$$s = a^p b^p c^p$$

case 1: vxy all one letter

uv^2xy^2z has more of that letter

case 2: v and y are each made up of a single letter

uv^2xy^2z has more of two of the letters

case 3: v or y contain multiple different symbols

$uv^2xy^2z \notin B$, bad ordering

aaabbbccc

u v x y z

$D = \{ww \mid w \in \{0,1\}^*\}$
is not a CFL

Choose $s = \underbrace{0^p 1^p}_{w}\underbrace{0^p 1^p}_{w}$.

split $s = uvxyz$ where $|vxy| \le p$

- vxy must cross the center of s

- so vxy is some 1's followed by some 0's
  uv^2xy^2z
  will have too many 1's in the first part or too many 0's in the second

$L =$ palindromes that contain an equal number of 0's and 1's

✗ 011010110

✗ 0110110110

✓ 01100110

$$S = 0^p 1^p 1^p 0^p$$

1: vxy is all 0's (from same 0^p)
more 0's than 1's

2: vxy is all 1's
more 1's than 0's

3. vxy are 0's and 1's
no longer a palindrome

L_1 : palindromes
L_2 : equal 0's and 1's

$$L = L_1 \cap L_2$$

CFLs not closed under $\cap$

Turing machines

Alan Turing
Alonzo Church

Church-Turing Thesis

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