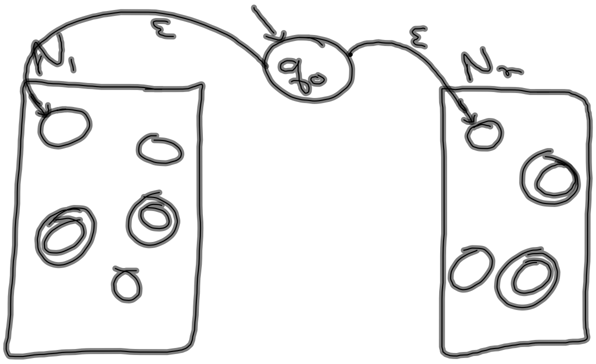
if A_1 and A_2 are regular
then $A_1 \cup A_2$ is regular.

N_1 is an NfA recognizing A_1
N_2 " " " " A_2



$$N_1\,(Q_1, \Sigma, \delta_1, q_1, F_1)$$
$$N_2\,(Q_2, \Sigma, \delta_2, q_2, F_2)$$

construct
$$N\,(Q, \Sigma, \delta, q_0, F)$$

$$Q = Q_1 \cup Q_2 \cup \{q_0\}$$

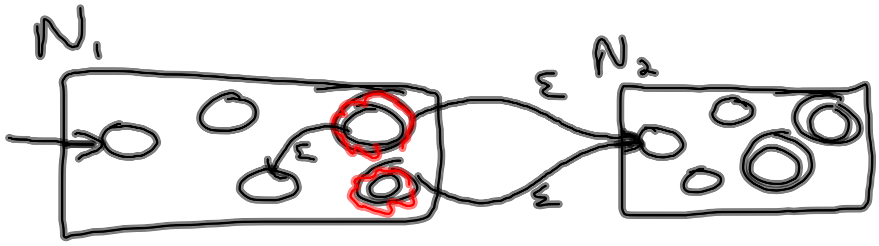
δ

$$\delta(q, a) = \begin{cases} \delta_1(q, a) & q \in Q_1 \\ \delta_2(q, a) & q \in Q_2 \\ \{q_1, q_2\} & q = q_0 \text{ and } a = \varepsilon \\ \{\ \} & q = q_0 \text{ and } a \neq \varepsilon \end{cases}$$

$q \in Q,\ a \in \Sigma_\varepsilon$

q_0
$$F = F_1 \cup F_2$$

if A_1 and A_2 are regular then $A_1 \circ A_2$ is regular



$A_1 = \{0, 1\}$

$A_2 = \{A, B\}$

$A_1 \circ A_2 = \{0B, 1B, 0A, 1A\}$

$$N_1(Q_1, \Sigma, \delta_1, q_1, F_1)$$
$$N_2(Q_2, \Sigma, \delta_2, q_2, F_2)$$
$$N(Q, \Sigma, \delta, \cancel{q_0}\ q_1, F_2)$$

$$Q = Q_1 \cup Q_2$$

$$\delta(q, a) = \begin{cases} \delta_1(q, a) & q \in Q_1 \text{ and } q \notin F_1 \\ \delta_2(q, a) & q \in Q_2 \\ \delta_1(q, a) & q \in F_1 \text{ and } a \neq \varepsilon \\ \{q_2\} \cup \delta_1(q, a) & q \in F \text{ and } a = \varepsilon \end{cases}$$

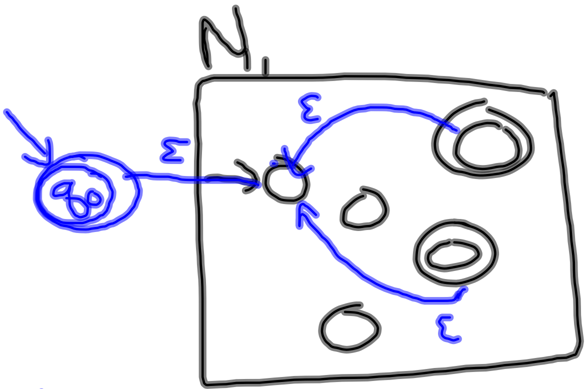
A^*

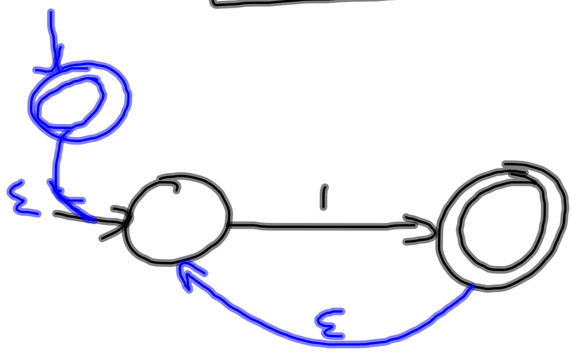
$A = \{0, 1\}$

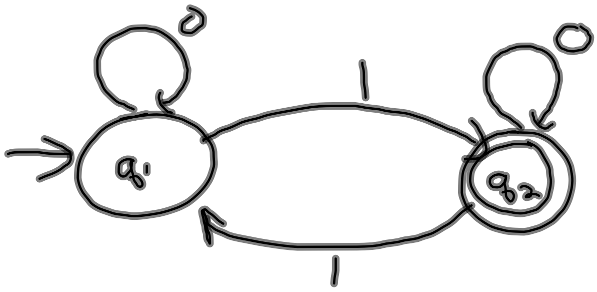
$A^* = \{\varepsilon, 0, 1,$
$00, 01, 10, 11$
$000, 001, 010,$
$\dots\}$

if A is regular
then A^* is regular

$\boxed{A^+ = A \circ A^*}$







$A = \{\text{odd \# of 1's}\}$

$A^* = \{\varepsilon, 11 \quad \}$

$00 \notin A^*$