A language is called <u>regular</u> iff some dfa recognizes it.

Let A and B be languages

Union: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

Concatenation: $A \circ B = \{xy \mid x \in A \text{ and } y \in B\}$

Star $A^* = \{x_1 x_2 x_3 \cdots x_k \mid k \geq 0 \text{ and each } x_i \in A\}$

$$\Sigma = \{a..z\}$$

$$A = \{one, two\}$$

$$B = \{fish, bird\}$$

$$A \cup B = \{one, two, fish, bird\}$$

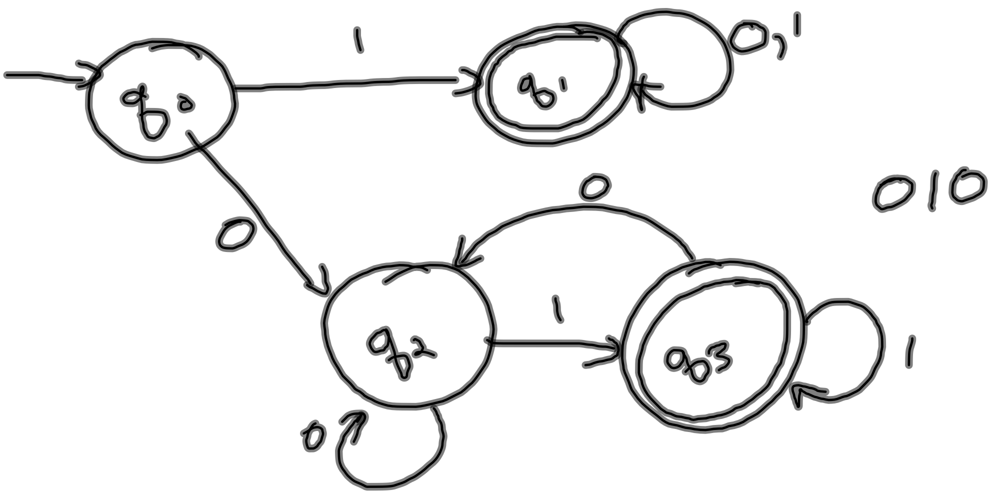
$$A \circ B = \{onefish, onebird, twofish, twobird\}$$

$$B \circ A = \{fishone, birdone, fishtwo, birdTwo\}$$

$$A^* = \{\epsilon, one, two, oneone, onetwo, twoone, twotwo, \ldots\}$$

DFA accepts all strings that begin w/ 1 or end w/ 1

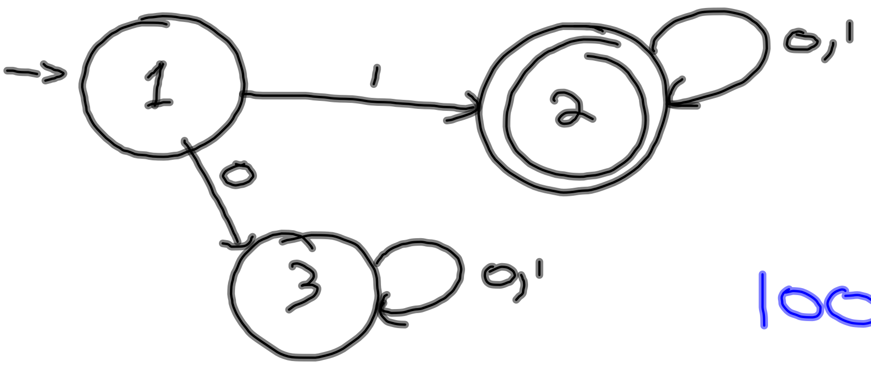
$\Sigma = \{0, 1\}$



$A_1 = \{ w \mid w \text{ begins w/ } 1 \}$

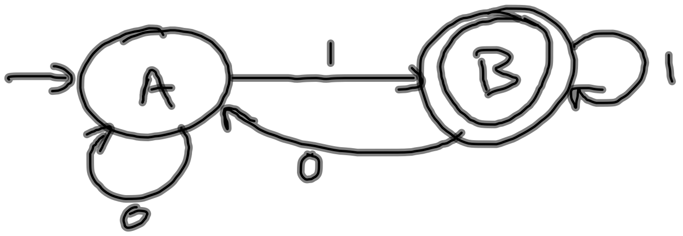
$A_2 = \{ w \mid w \text{ ends w/ } 1 \}$

M_1

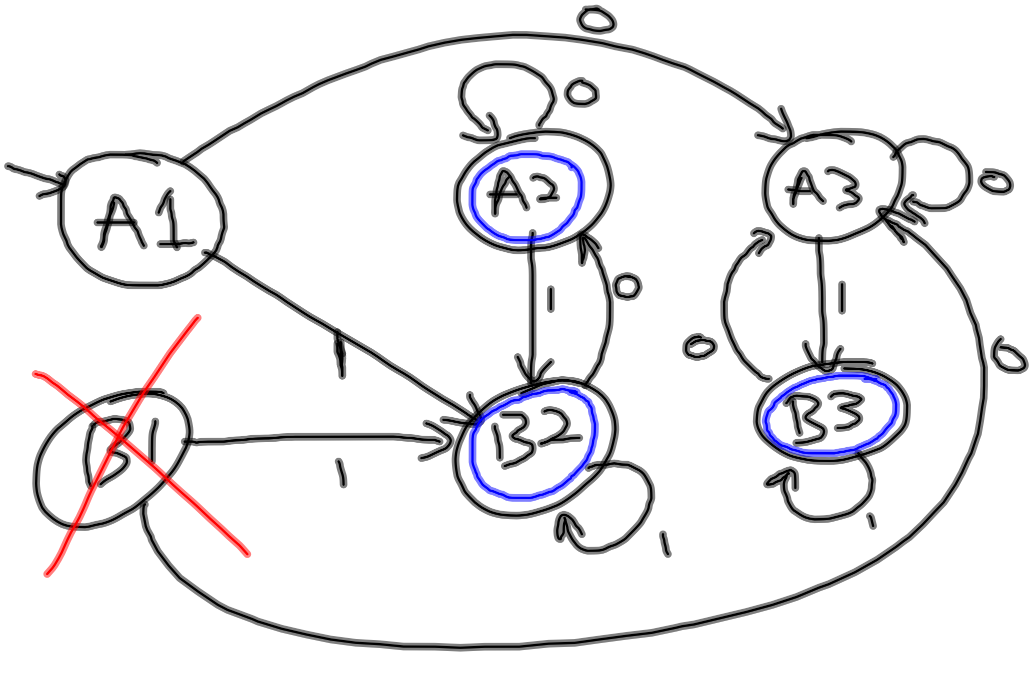


10010

M_2



$A_1 \cup A_2$



The class of regular languages is closed under the union operation.

if A_1 and A_2 are regular then $A_1 \cup A_2$ is regular

Proof:

suppose A_1 and A_2 are regular.
[we will show $A_1 \cup A_2$ is regular]

There is a dfa M_1 that accepts A_1 and M_2 that accept A_2

$$M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$$
$$M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$$

Construct M that recognizes $A_1 \cup A_2$

$$M = (Q, \Sigma, \delta, q_0, F)$$

$$Q = Q_1 \times Q_2 = \{(r_1, r_2) \mid r_1 \in Q_1 \text{ and } r_2 \in Q_2\}$$

Σ

δ: for each $(r_1, r_2) \in Q$, and $a \in \Sigma$

$$\delta((r_1, r_2), a) = (\delta_1(r_1, a), \delta_2(r_2, a))$$

$$q_0 = (q_1, q_2)$$

$$F = \{(r_1, r_2) \mid r_1 \in F_1 \text{ or } r_2 \in F_2\}$$

So M recognizes $A_1 \cup A_2$.
Therefore $A_1 \cup A_2$ is regular. //