Quotient -Remainder Theorem

Given an integer n and a positive integer d, there exists unique integers q, r such that

$n = dq + r$ and $0 \le r < d$

n div $d = q$

n mod $d = r$

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Integers are either even or odd.

Let n be an integer

by Q-R Thm. w/ $d = 2$

$n = 2q + r$ and $0 \le r < 2$

Consider $r = 0$

$n = 2q + 0 = 2q$

n is even

$r = 1$

$n = 2q + 1$

n is odd

Parity

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Consecutive integers have opposite parity.

Suppose we have two consecutive integers: m and $m+1$

By the parity property m is even or odd.

case 1: m is even

so $m = 2 \cdot k$ $k \in \mathbb{Z}$

and $m + 1 = 2k + 1$ subst.

$\therefore$ $m+1$ is odd by def. of odd.

case 2: m is odd

so $m = 2 \cdot r + 1$ $r \in \mathbb{Z}$

and $m + 1 = (2 \cdot r + 1) + 1$ subst.

$= 2r + 2$

$= 2(r + 1)$

Let $s = r + 1$, $s \in \mathbb{Z}$

by subst

$m + 1 = 2 \cdot s$

$\therefore$ $m+1$ is even

Concl: Two consec. integers have opposite parity.

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Division into cases

to prove $A_0 \vee A_1 \vee A_2 \vee \dots A_n \rightarrow C$

prove:

$$\left.\begin{array}{l} A_0 \rightarrow C \\ A_1 \rightarrow C \\ \vdots \\ A_n \rightarrow C \end{array}\right\} \text{cases}$$

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Given a real number x, the floor of x, $\lfloor x \rfloor$ is

floor $\lfloor x \rfloor$ = a unique integer n s.t. $n \le x < n+1$

ceiling $\lceil x \rceil = n \iff n - 1 < x \le n$

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For all real numbers x and all integers m, $\lfloor x + m \rfloor = \lfloor x \rfloor + m$

Proof:

Suppose x is a real number an m is an integer.

Let $n = \lfloor x \rfloor$ By def of floor $n \le$

$n \le x < n+1$

$n \le x < n + 1$

add m

$n + m \le x + m < n + 1 + m$

$n + m \le x + m < n + m + 1$

$(n+m) \in \mathbb{Z}$

By def. of floor

$n + m = \lfloor x + m \rfloor$

But $n = \lfloor x \rfloor$ so by subst.

$\lfloor x \rfloor + m = \lfloor x + m \rfloor$

QE!

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for all real numbers x, y

$$\lfloor x+y \rfloor = \lfloor x \rfloor + \lfloor y \rfloor$$

$x = y = 3.5$ ✓

$\lfloor x+y \rfloor = \lfloor 3.5+3.5 \rfloor = \lfloor 7 \rfloor = 7$

$\lfloor x \rfloor = \lfloor 3.5 \rfloor = 3$

$\lfloor y \rfloor = \lfloor 3.5 \rfloor = 3$

$\lfloor x \rfloor + \lfloor y \rfloor = 6$

$x = -5.2 \quad y = 3.7$ ✓

$\lfloor x+y \rfloor = \lfloor -5.2+3.7 \rfloor = \lfloor -1.5 \rfloor = -2$

$\lfloor x \rfloor = \lfloor -5.2 \rfloor = -6$

$\lfloor y \rfloor = \lfloor 3.7 \rfloor = 3$

$\lfloor x \rfloor + \lfloor y \rfloor = -3$

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