Prove: If m is even and n is odd then $m^2 + 3n$ is odd

Starting point: Suppose m is even and n is odd.

Show: $m^2 + 3n$ is odd.

Suppose m is even and n is odd

So m=2*s and n=2*r+1

where $s, r \in \mathbb{Z}$ by def. of even/odd

Now $m^2 + 3n = (2 \cdot s)^2 + 3(2 \cdot r + 1)$ by subst.

$= 4s^2 + 6r + 3$

$= (4s^2 + 6r + 2) + 1$

$= 2 \cdot (2s^2 + 3r + 1) + 1$

Let $k = 2s^2 + 3r + 1$

So $k \in \mathbb{Z}$ because it is a sum of products of integers.

$= 2 \cdot k + 1$ by subst.

We have shown that

$m^2 + 3n = 2k + 1, \ k \in \mathbb{Z}$

Therefore $m^2 + 3n$ is odd by def. of odd.

QED

Sep 24-10:40 AM

3.3 Divisiblity

If n and d are integers then n is divisible by d, iff $n = d \cdot k$ for some integer k.

n is a multiple of d

d is a factor of n

d is a divisor of n

d divides n

$d \mid n$

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if $n, d \in \mathbb{Z}$ and $d \neq 0$

$d \mid n \iff \exists k \in \mathbb{Z}$ s.t. $n = d \cdot k$

Sep 27-10:10 AM

For all integers a, b, c if $a \mid b$ and $b \mid c$, then $a \mid c$

Start: Let $a, b, c \in \mathbb{Z}$ and $a \mid b$ and $b \mid c$

Show: $a \mid c$

Since $a \mid b$, $b = am$, $m \in \mathbb{Z}$

and since $b \mid c$, $c = bn$, $n \in \mathbb{Z}$

$c = b \cdot n$

$= (a\,m)n$ by subst.

$= a(m\,n)$ assoc.

Let $k = m \cdot n$. Now $k \in \mathbb{Z}$ since it is the product of integers

So $c = a \cdot k$ for some integer k.

$\therefore a \mid c$

Sep 27-10:15 AM

Any integer $n > 1$ is divisible by a prime number.

Suppose $n \in \mathbb{Z}$ and $n > 1$

case 1: n is prime ✓

case 2: n is not prime

$n = r_0 s_0$ $r_0, s_0 \in \mathbb{Z}$

and $1 < r_0 < n$

$1 < s_0 < n$

By def of divisibility $r_0 \mid n$

case 1: r_0 is prime ✓ done

case 2: r_0 is not prime

$r_0 = r_1 s_1$ $r_1, s_1 \in \mathbb{Z}$

and $1 < r_1 < r_0$

$1 < s_1 < r_0$

⋮

?

Sep 27-10:24 AM

Unique Factorization Theorem for Integers (Fundamental Theorem of Arithmetic).

Given any integer $n > 1$, there exists a positive integer k and distinct prime numbers $p_1, p_2, \ldots p_k$ and pos. integers $e_1, e_2, \ldots e_k$ s.t.

$$n = p_1^{e_1} \cdot p_2^{e_2} \cdot p_3^{e_3} \cdot \ldots \cdot p_k^{e_k}$$

and any other expression of n as a product of prime numbers is identical to this except for the order in which they are written.

Sep 27-10:36 AM

Prove or give a counter example:

$\forall a, b \in \mathbb{Z}$ if $a \mid b$ and $b \mid a$ then $a = b$

Suppose $a, b \in \mathbb{Z}$ s.t. $a \mid b$ and $b \mid a$

$a \mid b$ $b = k \cdot a$ $k \in \mathbb{Z}$

$b \mid a$ $a = \ell b$ $\ell \in \mathbb{Z}$

So $b = k \cdot a$

$= k \cdot (\ell b)$

$= (k \cdot \ell) b$

so $k \cdot \ell = 1$

$k = \ell = 1$ $k = \ell = -1$

$b = ka = a$ $b = ka = -1 \cdot a = -a$

$b = -a$

counter example

$b = 3$

$a = -3$

Sep 27-10:42 AM