Show that the sum of two odd integers is even.

Formal Restatement: ∀ m,n in Z, if m and n are odd then m+n is even

Starting Point: Suppose m,n are particular but arbitrarily chosen integers that are odd.

Show: m+n is even.

Proof (formal)
Suppose m and n are odd integers. By the definition of odd, m = 2*r + 1 and n = 2*s + 1 for some integers r and s.

Then:

m+n	= 2*r + 1 + 2*s + 1	by substitution
	= 2*r + 2*s + 2	by algebra
	= 2*(r + s + 1)	by factoring out a 2

Let k = r + s + 1. Note that k is an integer because it is the sum of integers. Hence

m+n = 2*k where k is an integer

It follows from the definition of even that m+n is even.

QED

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Common mistakes (p135-137)
Tips (p 134-135)

QED: quod erat demonstrandum

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Rational numbers

r is rational $\Leftrightarrow$
$\exists\, a,b \in \mathbb{Z}$ s.t. $r = \frac{a}{b}$ and $b \neq 0$

$\forall x\, P(x) \leftrightarrow \forall y\, Q(y)$
$P(x) \Leftrightarrow Q(\cancel{x})$

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The sum of any two rational numbers is rational.

$\forall p,q,\ p \in \mathbb{Q}$ and $q \in \mathbb{Q} \rightarrow p+q \in \mathbb{Q}$

Starting point: Suppose p and q are particular but arbitrarily chosen rational numbers.

Show: p+q is rational

Suppose p,q are rationals.
Then $p = \frac{a}{b}$ and $q = \frac{c}{d}$ where $b \neq 0$ and $d \neq 0$ and a,b,c,d are integers by the definition of rational.

Thus $p+q = \frac{a}{b} + \frac{c}{d}$ by subst.
$= \frac{ad+cb}{bd}$ by algebra (common denom.)

Let $m = ad+cb$ and $n = bd$.
So m and n are integers since they are sums of products of integers.
Also $n \neq 0$ by the zero product property.
Therefore, $p+q = \frac{m}{n}$ where m,n are integers and $n \neq 0$.
So, by def. of rational p+q is rational.

QED

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Additional properties of even and odd integers (Example 3.2.3 p. 145)

1. The sum, product and difference of any two even integers are even.
2. The sum and difference of any two odd integers are even.
3. The product of any two odd integers is odd.
4. The product of any even integer and any odd integer is even.
5. The sum of any odd integer and any even integer is odd.
6. The difference of any odd integer minus any even integer is odd.
7. The difference of any even integer minus any odd integer is odd.

Prove: If m is even and n is odd then m^2 + 3n is odd

m^2 is even by prop. 1
3n is odd by prop. 3
m^2+3n is odd by prop. 5

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Prove: If m is even and n is odd then m^2 + 3n is odd

Starting point: Suppose m is even and n is odd.

Show m^2+3n is odd.

Suppose m is even and n is odd
So $m = 2\cdot s$ and $n = 2\cdot r+1$ where $s,r \in \mathbb{Z}$ by def. of even/odd

Now $m^2+3n = (2\cdot s)^2 + 3(2\cdot r+1)$ by Subst.

$m^2+3n = 2k+1,\ k \in \mathbb{Z}$

Therefore m^2+3n is odd by def. of odd.

QED

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