HW4 - due Monday 9/27

3.1 Direct Proof

$n \in \mathbb{Z}$ is even $\Leftrightarrow$
$\exists\, k \in \mathbb{Z}$ s.t. $n = 2k$

$n \in \mathbb{Z}$ is odd $\Leftrightarrow$
$\exists\, k \in \mathbb{Z}$ s.t. $n = 2k+1$

Sep 22-9:58 AM

for $n > 1$, $n \in \mathbb{Z}$

n is prime $\Leftrightarrow$ $\forall\, r,s \in \mathbb{Z}^+$
if $n = r \cdot s$ then
$r = 1$ or $s = 1$

n is composite $\Leftrightarrow$ $\exists\, r,s \in \mathbb{Z}^+$ s.t.
$n = r \cdot s$ and $r \neq 1$ and
$s \neq 1$

Sep 22-10:06 AM

Proving $\exists\, x \in D$ s.t. $Q(x)$

construction
1. give an example $Q(x)$ is true
2. give a set of directions for building x

Let $x =$ ______
show $Q(x)$ is true

Sep 22-10:10 AM

There exists an even int. n that can be written as the sum of two primes.

Let $n = 8$
$n = 5 + 3$

Sep 22-10:13 AM

Suppose r and s are integers.
Prove $\exists\, k \in \mathbb{Z}$ s.t. $22r + 18s = 2k$.

Let $k = 11r + 9s$
$k \in \mathbb{Z}$ ($\mathbb{Z}$ are closed under $+, \times$)

$2k = 2(11r + 9s)$ substitution
$= 22r + 18s$

Let $k = 11r + 9s$. Then k is an integer because it is the sum of products of integers. By substitution, $2 \cdot k = 2(11r + 9s)$ which equals $22r + 18s$ by the distributive law of algebra.

Sep 22-10:16 AM

Disproof of $\forall x \in D$, if $P(x) \to Q(x)$

show $\exists\, x \in D$ s.t. $P(x) \wedge \sim Q(x)$

statement: $\forall\, a,b \in \mathbb{R}$, if $a^2 = b^2$ then $a = b$
counterexample: Let $a = 2$, $b = -2$.
Then $a^2 = 4$ and $b^2 = 4$ so $a^2 = b^2$, but $a \neq b$ since $2 \neq -2$.

Sep 22-10:30 AM

Proving $\forall x \in D, P(x) \rightarrow Q(x)$

generalizing from a generic particular

1. Express the statement as: $\forall x \in D, P(x) \rightarrow Q(x)$
2. Suppose a is a particular but arbitrarily chosen element of D for which $P(a)$ is true.
3. Show $Q(a)$ is true.

Sep 22-10:36 AM

Show that the sum of two odd integers is even.

Restatement: $\forall r, s \in \mathbb{Z}$, if r,s are odd then r+s is even.

starting point: Suppose r,s are in $\mathbb{Z}$ that are odd

Show: r+s is even

proof:

r is odd

so $r = 2k+1$, $k \in \mathbb{Z}$

s is odd

so $s = 2t+1$, $t \in \mathbb{Z}$

$r+s = (2k+1)+(2t+1)$

$= 2k+2t+2$

$= 2(k+t+1)$

$k+t+1 \in \mathbb{Z}$

Let $j = k+t+1$

so $r+s = 2 \cdot j$, $j \in \mathbb{Z}$

def. of even

QED

Sep 22-10:43 AM