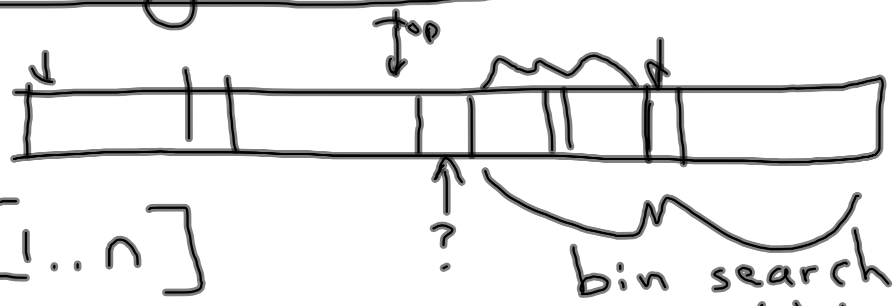
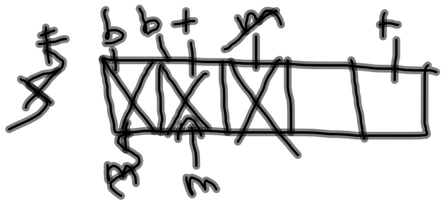
Binary Search






```
index = 0
bottom = 1
top = n
while (top ≥ bottom && index == 0) {
    mid = ⌊bottom + top / 2⌋;
    if (a[mid] == x)
        index = mid;
    else if (a[mid] > x)
        top = mid - 1;
    else
        bottom = mid + 1;
}
```

w_n : # of iterations of the loop in worst case execution of binary search

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$w_1 = 1$

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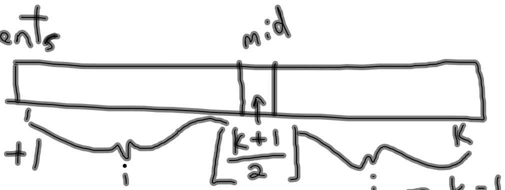
k - elements



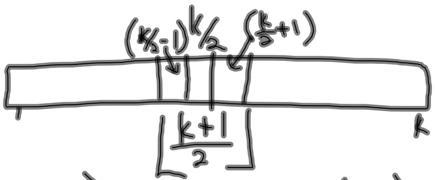
<u>k is odd</u>

$k = 2\cdot i + 1$

$i = \frac{k-1}{2}$

$\left\lfloor \frac{k}{2} \right\rfloor$ $\left\lfloor \frac{k}{2} \right\rfloor$

<u>k is even</u>



$\frac{k}{2} - 1 \cancel{-1+1}$ $k - \left(\frac{k}{2}+1\right) + 1$

$= \frac{k}{2}$

$w_1 = 1$

$w_k = 1 + w_{\lfloor \frac{k}{2} \rfloor}$

$w_2 = 1 + w_1 = 2$

$w_3 = 1 + w_1 = 2$

$w_4 = 1 + w_2 = 3$

$w_5 = 1 + w_2 = 3$

$w_6 = 1 + w_3 = 3$

$w_7 = 1 + w_3 = 3$

$w_8 = 1 + w_4 = 4$

$w_9 = 1 + w_4 = 4$

$w_{10} = 1 + w_5 = 4$

$\vdots$

$w_{15} = 1 + w_7 = 4$

$w_{16} = 1 + w_8 = 5$

$\vdots$

if $2^i \leq n < 2^{i+1}$

$w_i = i+1$ $\cancel{P}$

$k \in \mathbb{Z}, x \in \mathbb{R}$

and $2^k \leq x < 2^{k+1}$

then $\lfloor \log_2 x \rfloor = k$

(9.4.2)

$w_n = \lfloor \log_2 n \rfloor + 1$

$w_1 = 1$

$w_n = 1 + w_{\lfloor n/2 \rfloor}$ for $n > 1$

prove $w_n = \lfloor \log_2 n \rfloor + 1 \quad n \geq 1$

Strong induction:

Base case: suppose $n = 1$

$$w_1 = \lfloor \log_2 1 \rfloor + 1 = \lfloor 0 \rfloor + 1 = 1$$

Induction:
for all $k \geq 2$ if

$w_i = \lfloor \log_2 i \rfloor + 1$ for

$1 \leq i < k$ then

$w_k = \lfloor \log_2 k \rfloor + 1$

case 1: k is odd

$$\lfloor k/2 \rfloor = \frac{k-1}{2}$$

$$w_k = 1 + w_{\lfloor k/2 \rfloor} \qquad \text{def of } w$$

$$= 1 + w_{\frac{k-1}{2}}$$

$$= 1 + \left(\left\lfloor \log_2 \left(\frac{k-1}{2} \right) \right\rfloor + 1 \right)$$

$$= \lfloor \log_2 (k-1) - \cancel{\log_2 (2)}^{1} \rfloor + 2$$

$$= \lfloor \log_2 (k-1) \rfloor \cancel{-1+2} \; {+1}$$

for any odd int. $k > 1$

$\lfloor \log_2 (k-1) \rfloor = \lfloor \log_2 k \rfloor$

$$= \lfloor \log_2 (k) \rfloor + 1$$

~~woo hoo~~

QED