8.2 #7 $e_k = 4e_{k-1} + 5$ for $k \geq 1$

$e_0 = 2$

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$e_1 = 4(2) + 5$

$e_2 = 4(4(2)+5) + 5 = 4^2 \cdot 2 + 4 \cdot 5 + 5$

$e_3 = 4(4(4(2)+5)+5) + 5$

$= 4^3 \cdot 2 + 4^2 \cdot 5 + 4 \cdot 5 + 5$

$e_n = 4^n \cdot 2 + 5 \sum_{i=0}^{n-1} 4^i$

$$\boxed{\sum_{i=0}^{n} r^i = \left(\frac{r^{n+1} - 1}{r - 1}\right)}$$

$= 4^n \cdot 2 + 5\left(\frac{4^n - 1}{3}\right)$

$= 4^n \cdot 2 + \frac{5}{3} 4^n - \frac{5}{3}$

$= \frac{11}{3} 4^n - \frac{5}{3}$ for $n \geq 0$

8.2 # ~~7~~ 32 $e_k = 4e_{k-1} + 5$ for $k \geq 1$

$e_0 = 2$

prove:

$$e_n = \frac{11}{3}4^n - \frac{5}{3} \quad \text{for } n \geq 0$$

Basis: $e_0 = \frac{11}{3}4^0 - \frac{5}{3} = \frac{11}{3} - \frac{5}{3} = \frac{6}{3} = 2$

Induction: suppose

$$e_k = \frac{11}{3}4^k - \frac{5}{3}$$

show $e_{k+1} = \frac{11}{3}4^{k+1} - \frac{5}{3}$

$$e_{k+1} = 4 \cdot e_k + 5 \quad \text{def of } e$$

$$= 4\left(\frac{11}{3}4^k - \frac{5}{3}\right) + 5 \quad \text{ind. hyp.}$$

$$= \frac{11}{3}4^{k+1} - 4 \cdot \frac{5}{3} + 5$$

$$= \frac{11}{3}4^{k+1} \underbrace{- \frac{20}{3} + \frac{15}{3}}_{-\frac{5}{3}}$$

38b $\quad a_k = a_{k-1} + a_{k-3} + a_{k-4} + \cdots a_0 + 2$

show

$$a_k = 2a_{k-1} - a_{k-2} + a_{k-3}$$

$$a_k = a_{k-1} + a_{k-3} + \cancel{a_{k-4}} + \cancel{\cdots} + \cancel{a_0} + \cancel{2}$$

$$-\underbrace{(a_{k-2} + \cancel{a_{k-4}} + \cancel{a_{k-5}} + \cdots \cancel{a_0} + \cancel{2})}_{a_{k-1}}$$

$$a_k - a_{k-1} =$$

$$a_k - a_{k-1} = a_{k-1} - a_{k-2} + a_{k-3}$$

$$a_k = 2a_{k-1} - a_{k-2} + a_{k-3}$$

s=0;

for(int i=~~0~~1; i≤n; i++){ ← 1

  for(int j=~~0~~1; j≤i; j++){ ← 1

    s = s + j*(i-j+1); ← 4 ops

  }

}

$$\underset{\text{outer loop}}{n} + \overbrace{5 + 2\cdot 5 + 3\cdot 5 + \ldots + n\cdot 5}^{\text{inner loop}}$$

$$n + 5 * \sum_{i=1}^{n} i = n + 5\left(\frac{n(n+1)}{2}\right)$$

$$= n + 5\left(\frac{n^2+n}{2}\right)$$

$$= n + \frac{5}{2}n^2 + \frac{5}{2}n$$

$$= \frac{5}{2}n^2 + \frac{7}{2}n$$

$$\Theta(n^2)$$

assigns:

$$\underset{\text{int}}{2} + \underset{\substack{i,j \\ i{+}{+}}}{n} + 2\cdot\sum_{i=1}^{n} i$$

$$2 + n + \cancel{2}\left(\frac{n(n+1)}{\cancel{2}}\right)$$

$$2 + n + n^2 + n = 2 + 2n + n^2$$

$$\begin{array}{lr} & 2 + 2n + n^2 \\ + & \frac{7}{2}n + \frac{5}{2}n^2 \\ \hline & 2 + \frac{9}{2}n + \frac{7}{2}n^2 \end{array}$$

$$\Theta(n^2)$$