8.1 #38

a_k $\begin{cases} 0 \underline{\text{ str of length } k-1 \text{ w/o } 101} \\ \cancel{11} \underline{\quad " \quad k-2 \quad " \quad} \\ 100 \underline{\quad " \quad k-3 \quad " \quad} \\ 1100 \underline{\quad " \quad k-4 \quad " \quad} \\ 11100 \quad \vdots \end{cases}$

$a_3 = a_2 + a_0 + 2$

length 0 = ε $\quad a_0 = 1$

1 = 1, 0 $\quad a_1 = 2$

2 = 00, 01, 10, 11 $\quad a_2 = 4$

3 = 000, 001, 010, 011, 100, $\cancel{101}$, 110, 111 $\quad a_3 = 7$

$\begin{cases} 0 \underline{\text{ length } 2} & (4) \\ 100 \underline{\text{ len } 0} & (1) \\ 110 & \\ 111 & \end{cases}$

4 = 0000, 0001, 0010, 0011, 0100, $\cancel{0101}$, 0110, 0111, 1000, 1001, $\cancel{1010}$, $\cancel{1011}$, 1100, $\cancel{1101}$, 1110, 1111

0 $\underline{\ell = 3}$	7
100 $\underline{\ell = 1}$	2
1100 $\underline{\ell = 0}$	1
1110	
1111	

$f(x) = 2x^4 + 3x^3 + 5$

$g(x) = x^4$

$f(x)$ is $\Theta(g(x))$

1. $f(x)$ is $\Omega(g(x))$
2. show $f(x)$ is $O(g(x))$

$$2x^4 + 3x^3 + 5 \leq 2x^4 + 3x^4 + 5x^4$$

$$10 \cdot x^4$$

~~since~~ $x > 1$

we know $x^3 < x^4$

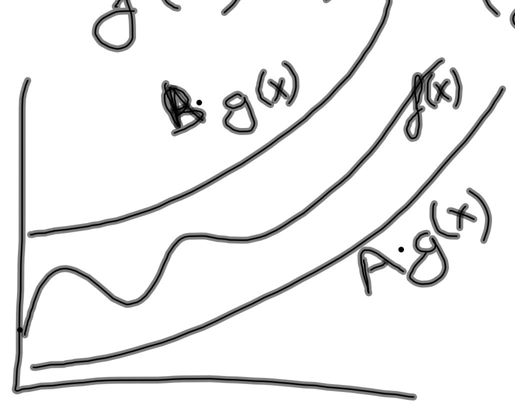
$1 < x^4$

$$|2x^4 + 3x^3 + 5| \leq 10|x^4|$$

w/ $b = 1$ and $B = 10$

by the def of O

$f(x)$ is $O(g(x))$



$a_0, \ldots a_n \in \mathbb{R}$ and $a_n \neq 0$

$a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$

is $O(x^s)$ $\forall s \geq n$

$\Omega(x^r)$ $\forall r \leq n$

$\Theta(x^n)$

$a_i, b_j \in \mathbb{R}$ $a_n \neq 0, b_m \neq 0$

$$\frac{a_n x^n + a_{n-1} x^{n-1} + \ldots a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \ldots b_1 x + b_0}$$

$\Theta(x^{n-m})$

$$\frac{5x^2 - 3}{4x}$$

$\Theta(x^{2-1})$

$\Theta(x)$

Efficiency of Algorithms

linear search
- comparisons
- size of data – n
- position of searched item

look at worst case
(not in array or
last item)

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matrix mult.

operations –

array access
additions
multiplications

```
p=0;
x=2;
for(int i=2; i≤n; i++){
    p=(p+1)·x;
}
```

$n-1$ iterations

1 add	(i++)
1 add	(p+1)
1 mul	$\cdot x$

$3(n-1)$ ops.

$$O(n)$$

also $\Theta(n)$, $\Omega(n)$