Permutations

An r-permutation of a set of n elements is an ordered selection of r elements taken from that set.

Set $\{a, b, c, d\}$

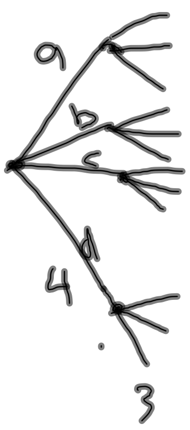
2-permutations

ab	ca
ac	cb
ad	cd
ba	da
bc	db
bd	dc

$$P(n, r) = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot (n-r+1)$$

$$= \frac{n!}{(n-r)!} = \frac{n(n-1)(n-2)\ldots(n-r+1)\cancel{(n-r)}\cancel{(n-r-1)}\cdot\ldots\cdot\cancel{2}\cdot\cancel{1}}{\cancel{(n-r)}\cancel{(n-r-1)}\cdot\ldots\cdot\cancel{2}\cdot\cancel{1}}$$

$$P(4, 2) = \frac{4 \cdot 3 \cdot \cancel{2} \cdot \cancel{1}}{\cancel{2} \cdot \cancel{1}}$$



$$P(\underset{n}{\overset{\text{obj}}{7}}, \underset{r}{\overset{\text{selections}}{4}}) = \frac{7 \cdot 6 \cdot 5 \cdot \overset{(n-r+1)}{4} \cdot \overset{(n-r)}{\cancel{3}} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$$

OBJECTS

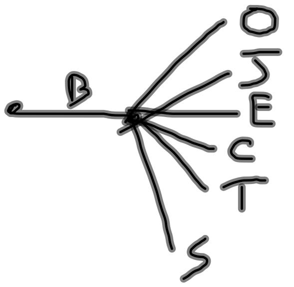
How many permutations of the letters?

$$7!$$

How many 3-permutations?

$$P(7,3) = 7 \cdot 6 \cdot 5 = 210$$

How many 3-permutations that begin w/ 'B'?



$$1 \cdot 6 \cdot 5 = 30$$

Number of passwords on a..z
w/ 3 or fewer letters

3 letter? 2 letter? 1 letter? 0 letter

26^3 26^2 26^1 $26^0 = 1$

$26 \cdot 26 \cdot 26$
a..z a..z
1st 2nd 3rd

3 or fewer

$26^3 + 26^2 + 26^1 + 26^0$

$= 18{,}279$

Addition rule: A is finite and a union of k mutually disjoint sets $A_1, \ldots A_k$ then

$$N(A) = N(A_1) + N(A_2) + \ldots + N(A_k)$$

(number of elements in A)

Integers (3 digit)
divisible by 5:

$$100, 101, 102, 103, 104, 105, \ldots, \underset{\uparrow}{995}, 999$$

$5 \cdot 20$ (100), $5 \cdot 21$ (105), $5 \cdot 199$ (995)

$$199 - 20 + 1 = \boxed{180}$$

Use addition rule

3-digit #'s ending in 5

$\_\ \_\ 5$

$1..9 \quad 0..9$

$$9 \cdot 10 \cdot 1 = 90$$

$\{ab5 \mid a \in \{1..9\},$ and $b \in \{0..9\}\}$

$\{n \mid n$ is a 3-dig # that ends in 5$\}$

ending in 0

$\_\ \_\ 0$

$$9 \cdot 10 \cdot 1 = 90$$

divisible by $5 = 90 + 90 = \boxed{180}$

A is a finite set and $B \subseteq A$

$$N(A - B) = N(A) - N(B)$$

Difference Rule

Passwords over $\{a..z\}$ of length 4 w/ repeats.

# of 4 char passwords

$$26^4$$

# of 4 char password w/o repeats

$$P(26,4) = 26 \cdot 25 \cdot 24 \cdot 23 = \frac{26!}{22!}$$

w/ repeats

$$26^4 - \frac{26!}{22!}$$