$$P_k = 2 \cdot P_{k-1} + 3^k \qquad k \geq 2$$

$$P_1 = 2$$

$$\vdots$$

$$P_n = 2^n + 2^n \left(\sum_{i=2}^{n} \left(\tfrac{3}{2}\right)^i \right) \qquad \boxed{\sum_{i=0}^{n} r^i = \left(\frac{r^{n+1}-1}{r-1}\right)}$$

$$= 2^n + 2^n \cdot \left[\sum_{i=0}^{n} \left(\tfrac{3}{2}\right)^i - \left(\tfrac{3}{2}\right)^0 - \left(\tfrac{3}{2}\right)^1 \right]$$

$$= 2^n + 2^n \cdot \left[\frac{\left(3/2\right)^{n+1} - 1}{\cancel{3/2 - 1}\ 1/2} \ \cancel{-1 - \tfrac{3}{2}}\ -5/2 \right]$$

$$= 2^n + 2^n \cdot \left[\cancel{2} \cdot \frac{3^{n+1}}{2^{\cancel{n+1}}} \ \cancel{-2 - \tfrac{5}{2}}\ -9/2 \right]$$

$$= 2^n + 3^{n+1} - \frac{9}{\cancel{2}} \cdot 2^{n-1}$$

$$= 2 \cdot 2^{n-1} + 3^{n+1} - 9 \cdot 2^{n-1} = 3^{n+1} - 7 \cdot 2^{n-1}$$

$p_k = 2 \cdot p_{k-1} + 3^k \quad k \geq 2$

$p_1 = 2$

show $p_n = 3^{n+1} - 7 \cdot 2^{n-1}$

Basis: $n = 1$

$p_1 = 3^{1+1} - 7 \cdot 2^{1-1} = 3^2 - 7 \cdot 2^0$

$= 9 - 7 = 2$

Induction: Suppose $p_k = 3^{k+1} - 7 \cdot 2^{k-1}$

Show $p_{k+1} = 3^{k+2} - 7 \cdot 2^k$

$p_{k+1} = 2 \cdot p_k + 3^{k+1}$ def of p

$= 2 \cdot \left[3^{k+1} - 7 \cdot 2^{k-1}\right] + 3^{k+1}$ Ind. Hyp.

$= 2 \cdot 3^{k+1} - 7 \cdot 2^k + 3^{k+1}$

$= 3 \cdot 3^{k+1} - 7 \cdot 2^k$

$= 3^{k+2} - 7 \cdot 2^k$

Ch. 9.

O, Ω, Θ Notations

↳ big O - Bachman

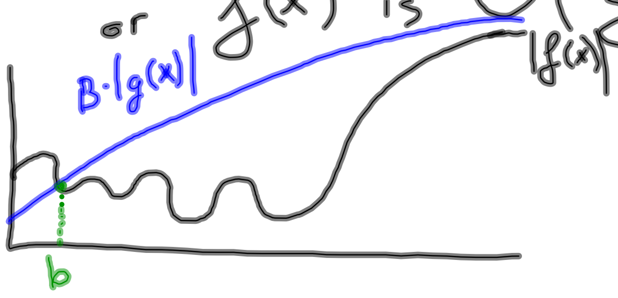
big omega } Knuth
big theta }

O : for functions f and g

If, for sufficiently large values of ~~x~~, the values of $|f(x)|$ are <u>less</u> than those of a multiple of $|g(x)|$

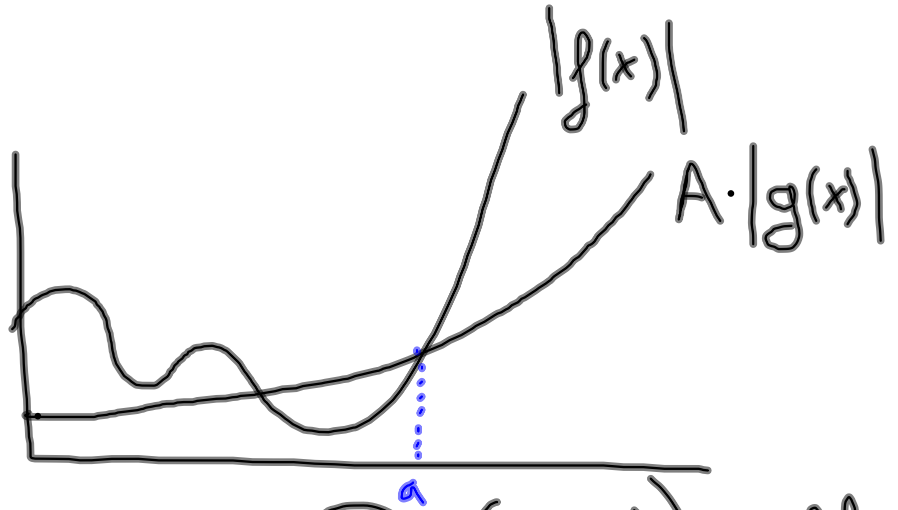
f is of order at <u>most</u> g

or $f(x)$ is $O(g(x))$



$f(x)$ is $O(g(x))$ iff there exists positive real numbers B and b such that $|f(x)| \leq B \cdot |g(x)|$ for all real numbers $x > b$

Ω

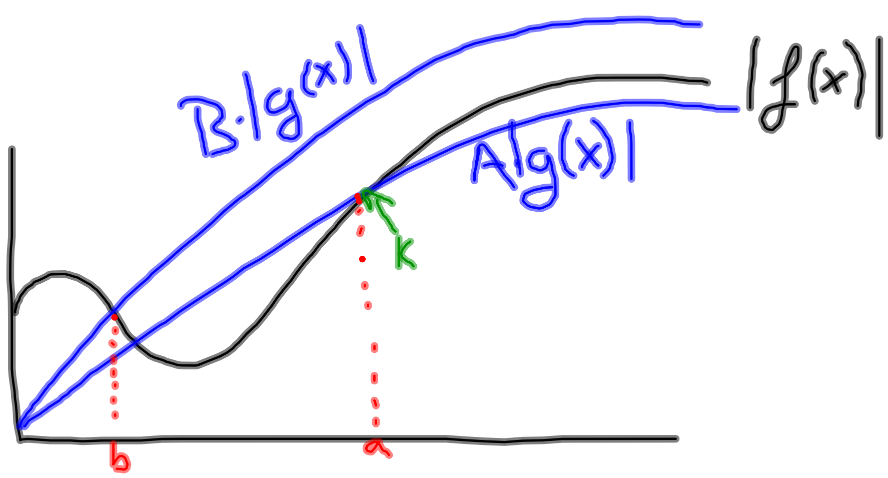


$f(x)$ is $\Omega(g(x))$ iff

$\exists\, A, a \in \mathbb{R}^+$ s.t.

$A \cdot |g(x)| \leq f(x)$ for all $x > a$

Θ



$f(x)$ is $\Theta(g(x))$ iff $\exists\, A, B, k \in \mathbb{R}^+$

s.t. $A\,|g(x)| \le |f(x)| \le B\,|g(x)|$

for all $x > k$

see
p.521

Show $2x^4 + 3x^3 + 5$ is $\Theta(x^4)$

$f(x)$ $\qquad\qquad\qquad\qquad$ $g(x)$

a. show $f(x)$ is $\Omega(g(x))$

$$2 \cdot x^4 \leq 2x^4 + \underbrace{3x^3 + 5}_{> 0 \text{ for } x > 0}$$

$$2|x^4| \leq |2x^4 + 3x^3 + 5|$$

w/ $A = 2$ and $a = 0$

$f(x)$ is $\Omega(g(x))$ by def of Ω