$S_{n,r}$ - set of size n into r subsets

$S_{4,3}$ $\{x_1, x_2, x_3, x_4\}$

$\{x_1, x_2\} \; \{x_3\} \; \boxed{\{x_4\}}$

$\{x_1, x_3\} \; \{x_2\} \; \boxed{\{x_4\}}$

$\{x_1, \boxed{x_4}\} \; \{x_2\} \; \{x_3\}$

$\{x_2, x_3\} \; \{x_1\} \; \boxed{\{x_4\}}$

$\{x_2, \boxed{x_4}\} \; \{x_1\} \; \{x_3\}$

$\{x_3, \boxed{x_4}\} \; \{x_1\} \; \{x_2\}$

added $\{x_4\}$ onto 3 elements into 2 subs. $S_{3,2}$

added to $S_{3,3}$

$\{x_1\} \; \{x_2\} \{x_3\}$

$\uparrow \; x_4 \quad \uparrow \; x_4 \quad \uparrow \; x_4$

$$S_{n,r} = S_{n-1,r-1} + r \cdot S_{n-1,r}$$

Base cases: $S_{n,n} = 1$ $\quad S_{n,1} = 1$

$a_k = a_{k-1} + 2 \quad k \geq 1$

$a_0 = 1$

$a_0 = 1$

$a_1 = 1 + 2$

$a_2 = (1+2) + 2$

$a_3 = ((1+2)+2)+2$

$a_4 = (1+2+2+2)+2$

$\vdots$

$a_n = 1 + 2n \quad$ for $n \geq 0$

Prove it!

Basis: $n=0$

$a_0 = 1 + 2 \cdot 0 = 1$

Induction:

Suppose $a_k = 1 + 2 \cdot k$

Show $a_{k+1} = 1 + 2(k+1) = 2k+3$

$a_{k+1} = a_k + 2$ by def of a

$= (1+2k)+2$ by Ind. Hyp.

$= 2k+3$ //

$\exists d$ s.t. $a_k = a_{k-1} + d$ for all $k \geq 1$

then $a_n = a_0 + d \cdot n \quad \forall n \geq 0$

(Arithmetic Sequence)

Geometric Sequence

$$a_k = r\, a_{k-1} \qquad r \neq 0$$

$$a_0 = a$$

$a_0 = a$

$a_1 = r \cdot a$

$a_2 = r(r \cdot a)$

$a_3 = r(r \cdot r \cdot a)$

$\vdots$

$a_n = r^n \cdot a$

Basis: $n=0$

$a_0 = r^0 \cdot a = a$

Ind.

Suppose $a_k = r^k \cdot a$

Show $a_{k+1} = r^{k+1} \cdot a$

$a_{k+1} = r \cdot a_k$

$= r \cdot r^k \cdot a$

$= r^{k+1} \cdot a$

$$p_k = 2\cdot p_{k-1} + 3^k \qquad k \geq 2$$

$$p_1 = 2$$

$$p_2 = 2(2) + 3^2$$

$$p_3 = 2\left(2(2) + 3^2\right) + 3^3 = 2\cdot2\cdot2 + 2\cdot3^2 + 3^3$$

$$p_4 = 2\left(2\left(2(2)+3^2\right) + 3^3\right) + 3^4$$

$$= \underbrace{\cancel{2\cdot2\cdot2\cdot2}}_{2^4} + 2\cdot2\cdot3^2 + 2\cdot3^3 + 3^4$$

$$p_5 = 2^5 + 2\cdot2\cdot2\cdot3^2 + 2\cdot2\cdot3^3 + 2\cdot3^4 + 3^5$$

$$= 2^5 + 2^3\cdot3^2 + 2^2\cdot3^3 + 2\cdot3^4 + 3^5$$

$$p_n = 2^n + \sum_{i=2}^{n} 2^{n-i}3^i = 2^n + \sum_{i=2}^{n} \frac{2^n}{2^i}3^i$$

$$= 2^n + 2^n\cdot\left(\sum_{i=2}^{n} \cancel{\frac{3^i}{2^i}}\right)$$

$$\left(\frac{3}{2}\right)^i$$

$$\sum_{i=0}^{n} r^i = \left(\frac{r^{n+1}-1}{r-1}\right)$$