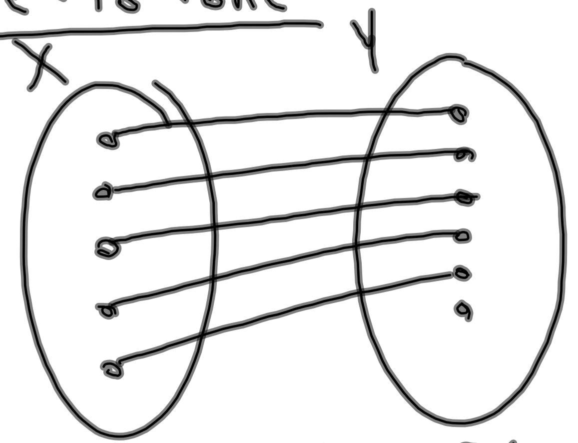
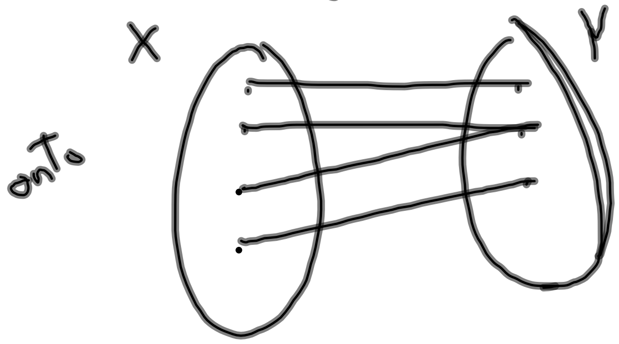
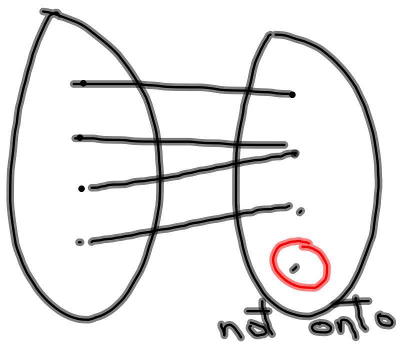
one-To-one



if $f(x_1) = f(x_2)$ then $x_1 = x_2$

onto (surjective)





every value $y \in Y$ has some $x \in X$
s.t. $y = F(x)$

$f : \mathbb{R} \to \mathbb{R}$

$f(x) = 4x - 1$

$\forall x \in \mathbb{R}$

is f onto?

Suppose $y \in \mathbb{R}$

find x s.t. $y = 4x - 1$

$x = \frac{y+1}{4}$

$x \in \mathbb{R}$

onto

$h : \mathbb{Z} \to \mathbb{Z}$

$h(n) = 4n - 1 \quad n \in \mathbb{Z}$

is h onto?

Let $y = 1$

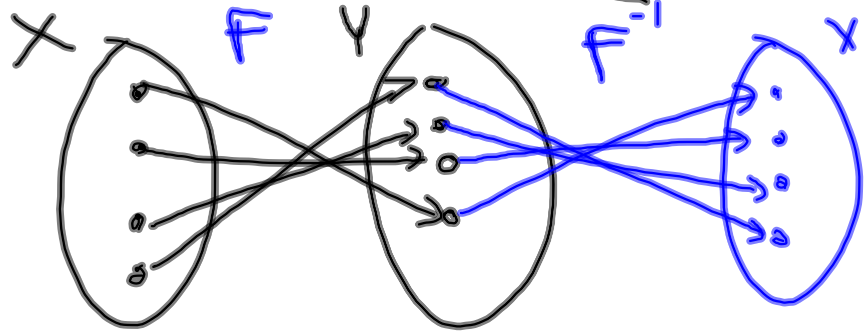
$y = 4n - 1$

$1 = 4n - 1$

$2 = 4n$

$n = \frac{1}{2} \notin \mathbb{Z}$

f is one to one correspondence (bijection) from X to Y
iff f is one to one and onto.



Suppose $f: X \to Y$ is a bijection

Then there is a function f^{-1}

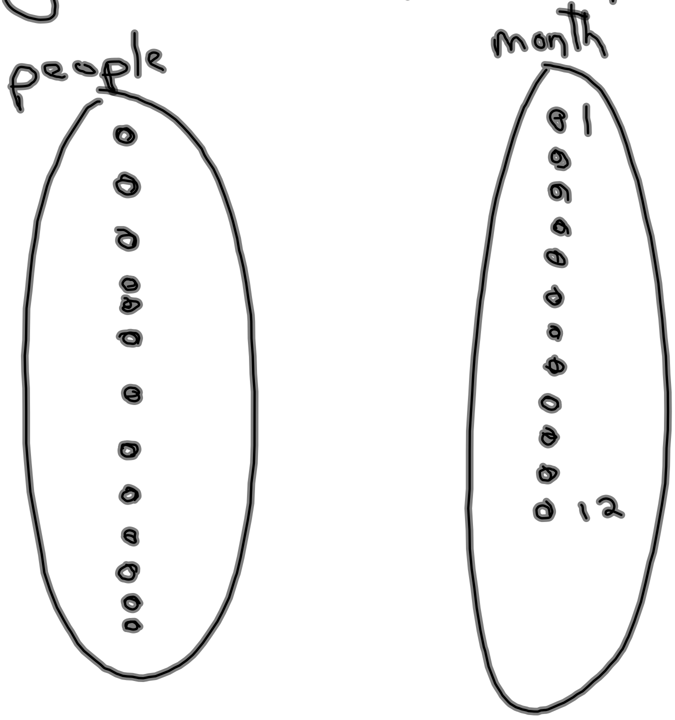
$$f^{-1}: Y \to X$$

Given any element $y \in Y$

$f^{-1}(y) =$ the unique $x \in X$ s.t. $f(x) = y$

so $f^{-1}(y) = x$ iff $y = f(x)$

Pigeon hole principle.

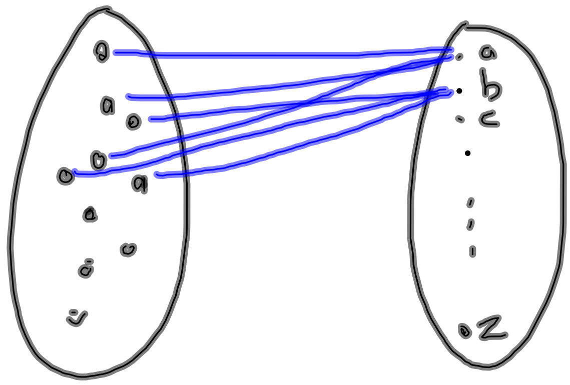


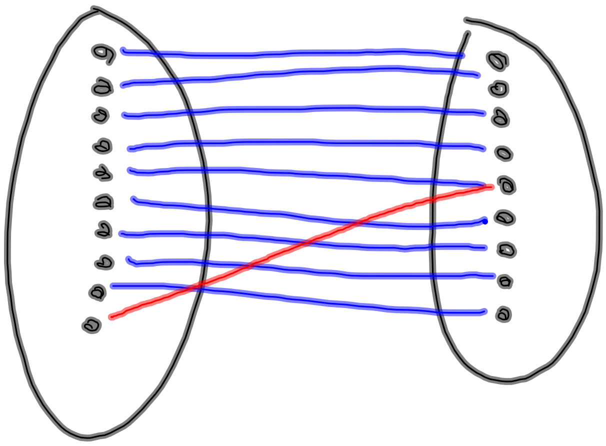
For any function $f: X \to Y$ and pos. int k

if $N(X) > k \cdot N(Y)$ then there is some y which is an image of <u>at least</u> $k+1$ distinct elements of X.

Group of 85 people, at least 4 must have the same last initial

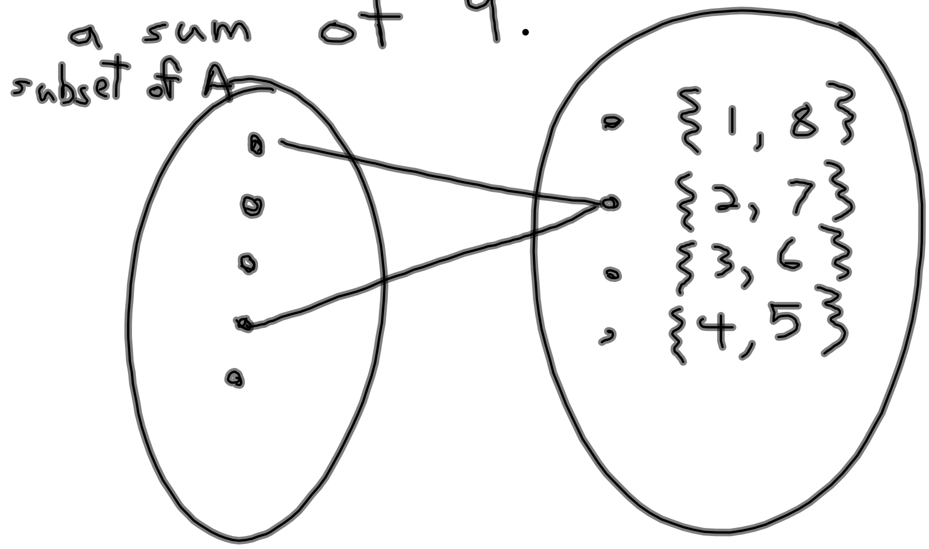
$$85 > 3 \cdot 26 = 78$$





$A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

If 5 integers are selected from A, must at least 1 pair have a sum of 9.



Contrapositive

For any function $f: X \to Y$
if for each $y \in Y$, y is
an image of k elements of X
then X has at most $k \cdot N(Y)$.

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