if $16 \mid n$ then $8 \mid n$

$$\sum_{i=1}^{n} (5i-4) = \frac{n(5n-3)}{2}$$

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Induction prove $P(k)$ $k \geq a$

Basis: $P(a)$

Induction if $P(k)$ then $P(k+1)$

Suppose $P(k)$

Show $P(k+1)$

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$$1+2+3+4+\ldots+n = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

Proof (by induction)

Basis: $n=1$?

$$\sum_{i=1}^{1} i = 1$$

$$\frac{1(1+1)}{2} = \frac{1(2)}{2} = 1$$

Left and right are both 1

Induction: Suppose it is true for $n=k$

$$\sum_{i=1}^{k} i = \frac{k(k+1)}{2} \quad \text{for some } k \geq 1$$

$(k+1)+1$

We must show that $\sum_{i=1}^{k+1} i = \frac{(k+1)(k+2)}{2}$

$$\sum_{i=1}^{k+1} i = \sum_{i=1}^{k} i + (k+1)$$ separate final term

$1+2+3+\ldots+k+(k+1)$

$$= \left(\frac{k(k+1)}{2}\right) + (k+1)$$ substitution by inductive hypothesis

$$= \frac{k(k+1)}{2} + \frac{2(k+1)}{2}$$

$$= \frac{k(k+1)+2(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2}$$ factor out $k+1$

Which is what we were trying to show

QED

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Prove $\prod_{i=2}^{n}\left(1-\frac{1}{i^2}\right) = \frac{n+1}{2n}$ for all integers $n \geq 2$

Basis: $n=2$

$$1-\frac{1}{2^2} = 1-\frac{1}{4} = \frac{3}{4}$$

$$\frac{n+1}{2n} = \frac{2+1}{2(2)} = \frac{3}{4}$$

Induction

Suppose $\prod_{i=2}^{k}\left(1-\frac{1}{i^2}\right) = \frac{k+1}{2k}$

Show $\prod_{i=2}^{k+1}\left(1-\frac{1}{i^2}\right) = \frac{(k+1)+1}{2(k+1)}$

$$\prod_{i=2}^{k+1}\left(1-\frac{1}{i^2}\right) = \left[\prod_{i=2}^{k}\left(1-\frac{1}{i^2}\right)\right]\cdot\left(1-\frac{1}{(k+1)^2}\right)$$ sep. last term

$$= \left(\frac{k+1}{2k}\right)\left(1-\frac{1}{(k+1)^2}\right)$$ sub. induction hypothesis

$$= \frac{k+1}{2k} - \frac{\cancel{(k+1)}}{2k}\cdot\frac{1}{(k+1)^{\cancel{2}}}$$

$$= \frac{k+1}{2k} - \frac{1}{2k(k+1)}$$

$$= \frac{(k+1)(k+1)-1}{2k(k+1)}$$

$$= \frac{(k^2+2k+1)-1}{2k(k+1)} = \frac{k^2+2k}{2k(k+1)} = \frac{\cancel{k}(k+2)}{2\cancel{k}(k+1)}$$

$$= \frac{k+2}{2(k+1)} = \frac{k+1+1}{2(k+1)}$$

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