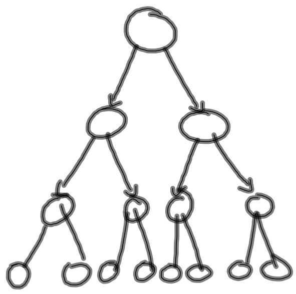
Level	# nodes
1	1
2	2
3	4
4	8



Sequence: 1, 2, 4, 8, 16, ...

$a_1, a_2, a_3, a_4, a_5, \dots$

Nodes at level k $\quad A_k = 2^{(k-1)}$

for $k \geq 1$

Explicit formula

general "

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Alternating sequence

$c_j = (-1)^j$ for $j \geq 0$

$c_0, c_1, c_2, c_3 \dots$

$1, -1, 1, -1,$

$1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \frac{1}{25}, -\frac{1}{36}, \dots$

$a_k = (-1)^{(k-1)} \frac{1}{k^2} \qquad k \geq 1$

$= \frac{(-1)^{k-1}}{k^2}$

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Series

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \dots + a_n$$

upper limit

lower limit

e.g. $\sum_{k=0}^{3} 2^k = 2^0 + 2^1 + 2^2 + 2^3$

$= 1 + 2 + 4 + 8$

$= 15$

$$\sum_{k=m}^{n} a_k = \left(\sum_{k=m}^{n-1} a_k\right) + a_n$$

or

$$= a_m + \left(\sum_{k=m+1}^{n} a_k\right)$$

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$$\frac{1}{k} - \frac{1}{k+1} = \frac{(k+1) - k}{k(k+1)} = \frac{1}{k(k+1)}$$

$$\sum_{k=1}^{n} \frac{1}{k(k+1)} = \left(\frac{1}{1} - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right)$$

$$+ \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{5}}\right)$$

$$+ \dots + \left(\cancel{\frac{1}{n}} - \frac{1}{n+1}\right)$$

$$= 1 - \frac{1}{n+1}$$

Telescoping Sum

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Products

$$\prod_{k=1}^{n} a_k = a_1 \cdot a_2 \cdot a_3 \cdot \dots \cdot a_n$$

$$\prod_{k=1}^{n} k = 1 \cdot 2 \cdot 3 \cdot \dots \cdot n = n!$$

factorial $\quad 0! = 1$

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$$\sum_{k=m}^{n} a_k + \sum_{k=m}^{n} b_k = \sum_{k=m}^{n} (a_k + b_k)$$

$$c \cdot \sum_{k=m}^{n} a_k = \sum_{k=m}^{n} c \cdot a_k$$

$$\prod_{k=m}^{n} a_k \cdot \prod_{k=m}^{n} b_k = \prod_{k=m}^{n} (a_k \cdot b_k)$$

$$\sum_{k=1}^{4} 2^{(k-1)} = 2^0 + 2^1 + 2^2 + 2^3$$

$$= \sum_{j=0}^{3} 2^j \quad \text{change of variable}$$

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Induction

Let $P(n)$ be some property on integers n

Suppose

1. $P(a)$ is true
2. for all integers $k \geq a$
   if $P(k)$ is true then $P(k+1)$ is true

Conclude that for all integers $n \geq a$ $P(n)$ is true

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Method: proof by induction

statements: for all integers $n \geq a$ $P(n)$ is true

Step 1: Show the property is true for $n=a$ (Basis)

Step 2: (inductive) Show: for all $k \geq a$ if $P(k)$ then $P(k+1)$

Suppose $P(k)$ is true
Show $P(k+1)$ is true

$P(k)$ inductive hypothesis

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