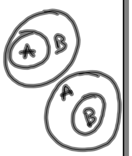
Prove $A \subseteq B$

if $x \in A$ then $x \in B$

$A = B$

I. $A \subseteq B$

II. $B \subseteq A$



$A = \emptyset$

suppose $A \neq \emptyset$ i.e. $\exists\, x \in A$

show a contradiction

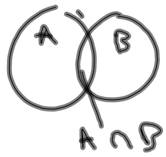
Oct 27-10:00 AM

Prove $A \cap B \subseteq A$

suppose $x \in (A \cap B)$

so $x \in A$ and $x \in B$
by def. of $\cap$

so $x \in A$



Oct 27-10:10 AM

prove $(A-B) \cup (C-B) = (A \cup C) - B$
for all sets A, B, C

I. show $(A-B) \cup (C-B) \subseteq (A \cup C) - B$

II. show $(A \cup C) - B \subseteq (A-B) \cup (C-B)$

I. suppose $x \in (A-B) \cup (C-B)$

so i $\boxed{x \in (A-B)}$ or $\boxed{x \in (C-B)}$ ii

i. show if $x \in (A-B)$ then $x \in (A \cup C) - B$

suppose $x \in (A-B)$

so $x \in A$ and $x \notin B$

so $x \in (A \cup C)$ by def of $\cup$

since $x \in (A \cup C)$ and $x \notin B$

we know $x \in (A \cup C) - B$
by def of $-$

ii. similar to i

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II. show $(A \cup C) - B \subseteq (A-B) \cup (C-B)$

suppose $x \in (A \cup C) - B$

$x \in (A \cup C)$ and $x \notin B$ by def of $-$

so $x \in A$ or $x \in C$

i. $x \in A$ and $x \notin B$

If $x \in A$ then show $x \in (A-B) \cup (C-B)$

so $x \in (A-B)$ by def of $-$

and so $x \in (A-B) \cup (C-B)$ by def of $\cup$

ii. $x \in C$ and $x \notin B$

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$\forall$ sets A, B, C

if $A \subseteq B$ and $B \subseteq C^c$

then $A \cap C = \emptyset$

Suppose $A \cap C$ is not empty

so $x \in A \cap C$

by def of $\cap$ $x \in A$ and $x \in C$

Since $A \subseteq B$ x must also be in B

since $B \subseteq C^c$ x must also be in C^c

$x \in C^c$ and $x \in C$

which is a contradiction

so x does not exist.

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$(A^c)^c = A$

$A \cup A = A$

$(A \cup B)^c = \underline{A^c} \cap \underline{B^c}$

$(A \cap B)^c = A^c \cup B^c$

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$(A \cup B) - C = (A - C) \cup (B - C)$

$(A \cup B) - C = (A \cup B) \cap C^c$ set diff. law

$= (A \cap C^c) \cup (B \cap C^c)$ dist. law

$= (A - C) \cup (B - C)$ set diff. law

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Russell's Paradox

$S = \{ A \mid A$ is a set and $A \notin A \}$

so $S \in S$?

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