$e_0=1$, $e_1=2$, $e_2=3$

$e_k = e_{k-1}+e_{k-2}+e_{k-3}$ $k\geq 3$

$e_3=e_2+e_1+e_0 = 3+2+1=6$

$e_4 = e_3+e_2+e_1 = 6+3+2=11$

Prove: $e_n \leq 3^n$ for $n\geq 0$

Basis: show for $n=0,1,2$

$e_0=1 \leq 3^0=1$

$e_1=2 \leq 3^1=3$

$e_2=3 \leq 3^2=9$

Induction:

Suppose $e_k\leq 3^k$ for $0\leq k\leq n$

Show $e_{k+1}\leq 3^{k+1}$

weak? $P(k)\rightarrow P(k+1)$

$P(0)\wedge P(1)\wedge\ldots P(k)\rightarrow P(k+1)$ strong

$e_{k+1} = e_k+e_{k-1}+e_{k-2}$ def of k+1 element of e

$\leq 3^k+e_{k-1}+e_{k-2}$ Ind Hyp

$\leq 3^k+3^{k-1}+3^{k-2}$

$\leq 3^k+3^k+3^{k-2}$

$\leq 3^k+3^k+3^k$

$=3\cdot 3^k$

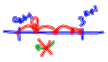
$=3^{k+1}$

$a=b\leq c\leq d$

$a\leq d$

$a\leq c$

QED



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$f_1=1$, $f_k=2\cdot f_{\lfloor k/2\rfloor}$ $k\geq 2$

Prove $f_n\leq n$ for $n\geq 1$

2x 2x 2x

1, 2, 2, 4, 4, 4,

subscr: 1 2 3 4 5 6

$f_3=2\cdot f_{\lfloor 3/2\rfloor}$

$=2\cdot f_{\lfloor 1.5\rfloor}$

$=2\cdot f_1$

$f_6=2\cdot f_{\lfloor 6/2\rfloor}$

$=2\cdot f_3$

$f_n\leq n$ $n\geq 1$

Basis $n=1$

$f_1=1=n$

Ind. Suppose $f_k\leq k$ for $1\leq k\leq$

Show $f_{k+1}\leq k+1$

$f_{k+1} = 2\cdot f_{\lfloor\frac{k+1}{2}\rfloor}$ $f_m\leq$

$\leq 2\left(\left\lfloor\frac{k+1}{2}\right\rfloor\right)$

$\leq \cancel{2}\cdot\left(\frac{k+1}{\cancel{2}}\right) = k+1$ //

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