## Indirect Proof

Proof by contradiction

$\forall x \in D,\ P(x) \to Q(x)$

Starting point: Suppose the negation is true.

$\exists x \in D$ s.t. $P(x) \wedge \sim Q(x)$

Show: a contradiction (often $\sim P(x)$ or $Q(x)$)

conclude the original statement is true.

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There is no greatest integer.

Proof (by contradiction)

Suppose there is a greatest integer; call it N.

So $N \geq n$ for every $n \in \mathbb{Z}$

Let $M = N + 1$

Then $M > N$ and $M \in \mathbb{Z}$ (sum of integers)

Thus M is an integer greater than N which is a contradiction.

Therefore, there is no largest integer.

$\blacksquare$

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$\forall x \in D,\ P(x) \to Q(x)$

Proof by contrapositive

$\forall x \in D,\ \sim Q(x) \to \sim P(x)$

Direct Proof

Suppose $\sim Q(x)$

Show $\sim P(x)$

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$\forall a,b,c \in \mathbb{Z}\quad a \nmid bc \to a \nmid b$

contrapos.

$\forall a,b,c \in \mathbb{Z}\quad a \mid b \to a \mid bc$

Suppose $a \mid b$ where $a,b \in \mathbb{Z}$

Show $a \mid bc$ where $a,b,c \in \mathbb{Z}$

Proof:

Suppose $a \mid b$ where $a,b \in \mathbb{Z}$

By def. of divides $b = a \cdot k$ $k \in \mathbb{Z}$

So $bc = (ak) \cdot c$ by subst

$= a \cdot (k \cdot c)$

Let $r = k \cdot c$ ; $r \in \mathbb{Z}$ prod. of integ

So $bc = ar$ where $r \in \mathbb{Z}$ (subst)

Therefore $a \mid bc$ by def of $\mid$

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$\forall a,b,c \in \mathbb{Z}\quad a \nmid bc \to a \nmid b$

Proof by contradiction

Suppose the negation is true.

$\exists a,b,c \in \mathbb{Z}$ st. $a \nmid bc \wedge a \mid b$

Since $a \mid b$, $b = k \cdot a$ $k \in \mathbb{Z}$

by subst $bc = (k \cdot a)c$ b

$= c \cdot k \cdot a$

Let $r = ck$ so $r \in \mathbb{Z}$

So $bc = r \cdot a$ where $r \in \mathbb{Z}$

So $a \mid bc$ by def of $\mid$

which is a contradiction

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$\forall n \in \mathbb{Z}$, if $n^2$ is odd then n is odd.

Proof by contradiction:
Suppose the negation is true. In other words, there is an integer **n** such that **n**^2 is odd, but n is not odd.

Since n is not odd, it is even so it can be written as n=2*k where k is an integer.

By substitution, n^2 = (2*k)^2 = 4*k^2 = 2*(2*k^2)

Let m = 2*k^2. Now m is an integer (closure). So n^2 = 2*m

By def of even, n^2 is even, which is a contradiction.

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