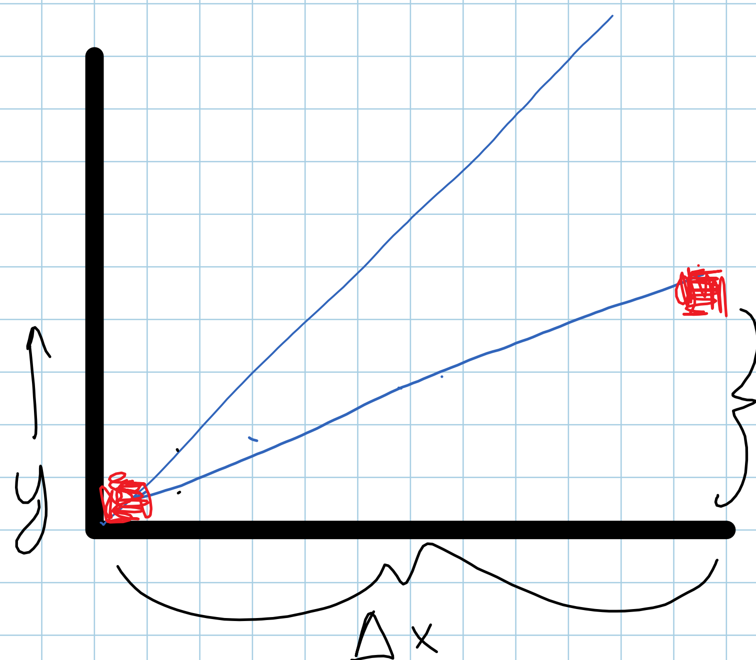
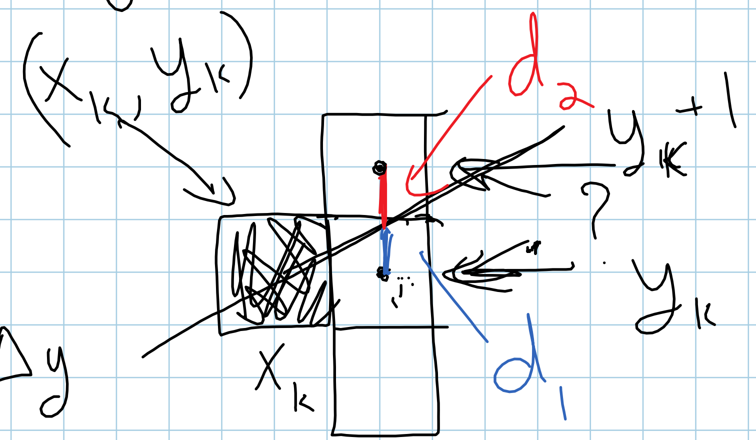
Bresenham's line algorithm





$$x_{k+1} = x_k + 1$$

next value on the line

$$y = m(x_k + 1) + b$$

$$m = \frac{\Delta y}{\Delta x}$$

$$d_1 = y - y_k = m(x_k + 1) + b - y_k$$

$$d_2 = y_k + 1 - y = y_{k+1} - m(x_k + 1) - b$$

consider $d_1 - d_2 = \begin{cases} < 0 & d_1 \text{ is small} \\ > 0 & d_2 \text{ is small} \end{cases}$

compute $\Delta x (d_1 - d_2) =$

$$2\Delta y\, x_k - 2\Delta x\, y_k + \underbrace{2\Delta y + \Delta x(2b-1)}_{\text{constant } c}$$

Decision param at step k $\boxed{p_k}$

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at step $k+1$

$$p_{k+1} = 2\Delta y \, x_{k+1} - 2\Delta x \cdot y_{k+1} + C$$

$$p_{k+1} - p_k = 2\Delta y \underbrace{(x_{k+1} - x_k)}_{1} - 2\Delta x \underbrace{(y_{k+1} - y_k)}_{\substack{0 \text{ or } 1 \\ \text{dep. on } p_k}}$$

$$p_{k+1} = p_k + 2\Delta y - 2\Delta x (y_{k+1} - y_k)$$

$$p_0 = 2\Delta y - \Delta x$$

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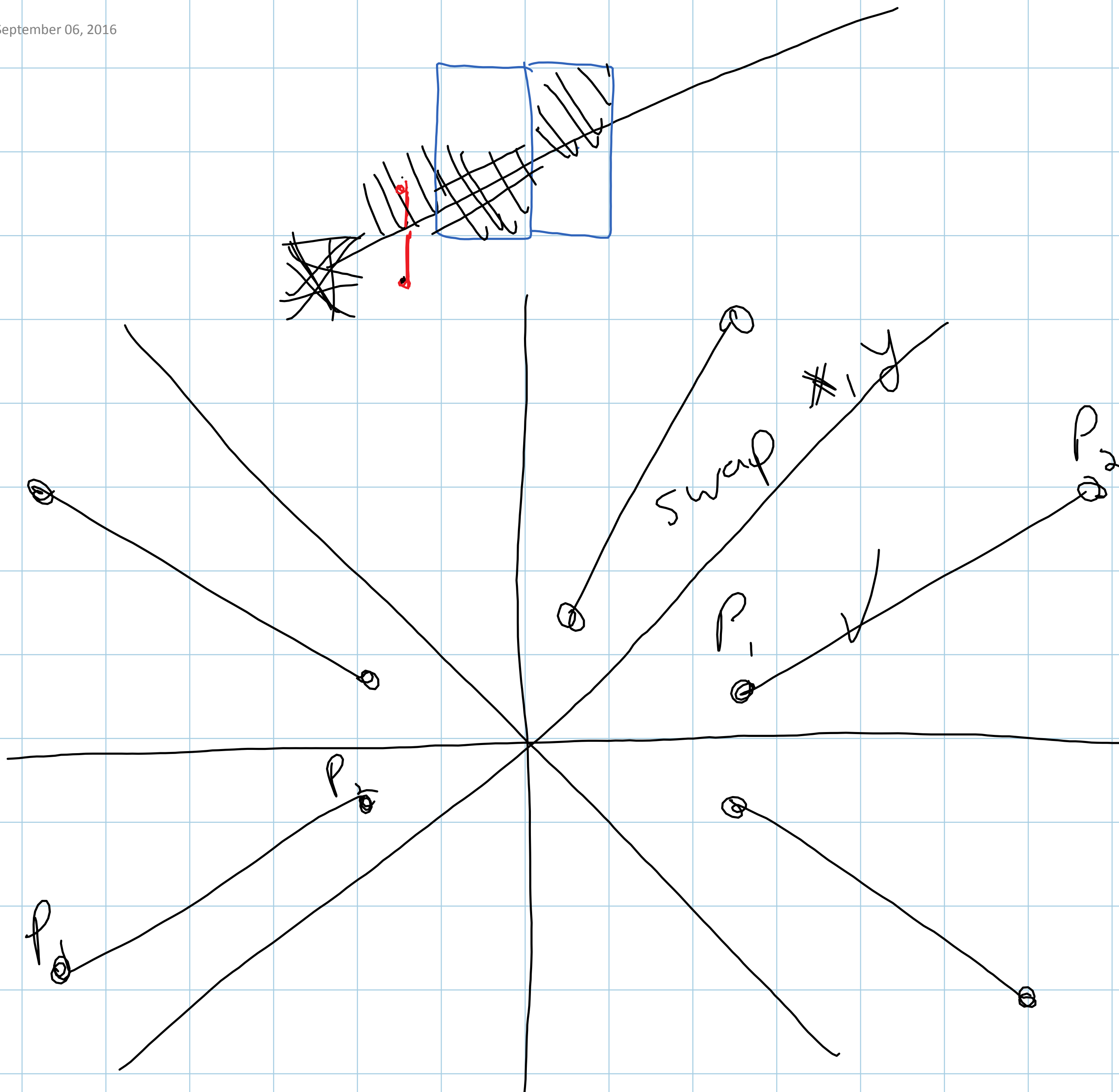
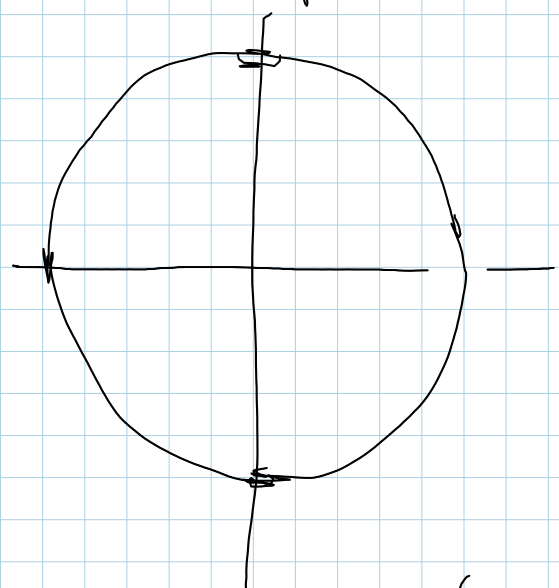
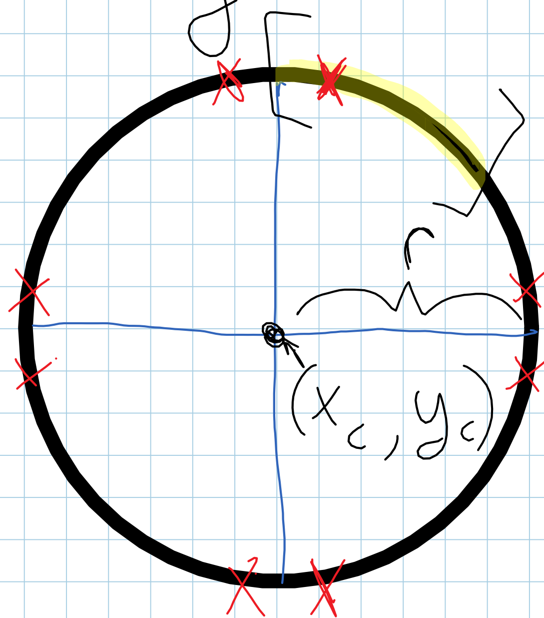
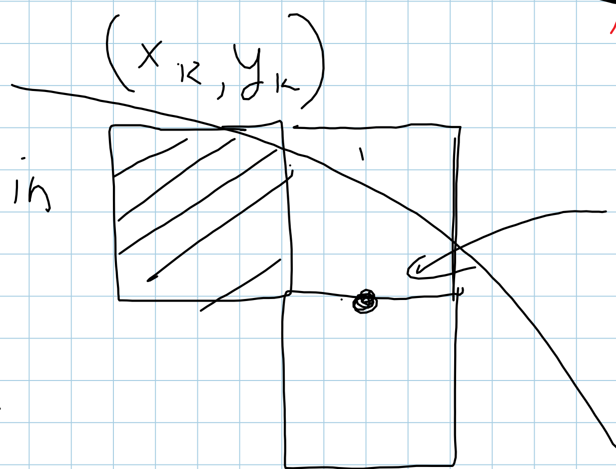


Mid-point circle algorithm





Is mid-pt in or out of circle?



mid point between y_k and $y_k - 1$

$$f_{circ}(x, y) = x^2 + y^2 - r^2$$

$$f_{circ}(x, y) = \begin{cases} 0 & \text{if } (x, y) \text{ is } \underline{\text{on}} \\ <0 & \text{if } (x, y) \text{ is } \underline{\text{in}} \\ >0 & \text{if } (x, y) \text{ is } \underline{\text{out}} \end{cases}$$

Deciding between (x_{k+1}, y_k)

and $(x_{k+1}, y_k - 1)$

$$x_{k+1} = x_k + 1$$

Dec. pt $\quad p_k = f_{circ}\left(x_k + 1, y_k - \frac{1}{2}\right)$

$$p_{k+1} = p_k + 2(x_k + 1) + (y_{k+1}^2 - y_k^2) - (y_{k+1} - y_k) + 1$$

y_{k+1} is y_k or $y_k - 1$

$$p_{k+1} = p_k + 2x_{k+1} + 1 \quad \text{if } p_k < 0$$

$$\text{or } p_k + 2x_{k+1} + 1 - 2y_{k+1} \quad \text{if } p_k > 0$$

$$p_0 = f_{circ}\left(1, r - \frac{1}{2}\right)$$

$$= \frac{5}{4} - r$$

for an integer r

$$p_0 = 1 - r$$